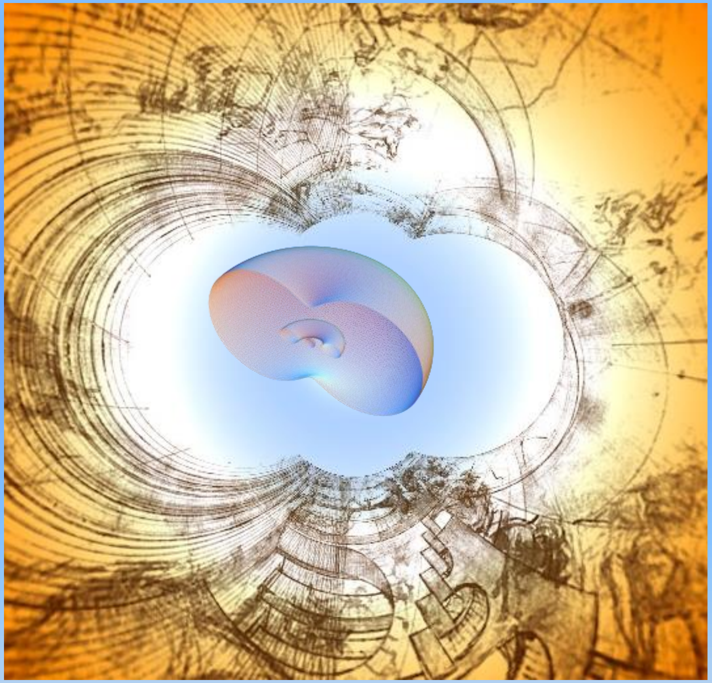
Astronomical Institute of the Romanian Academy
Bucuresti, 28.02.2024

Black holes, scalar fields and no-hair conjecture



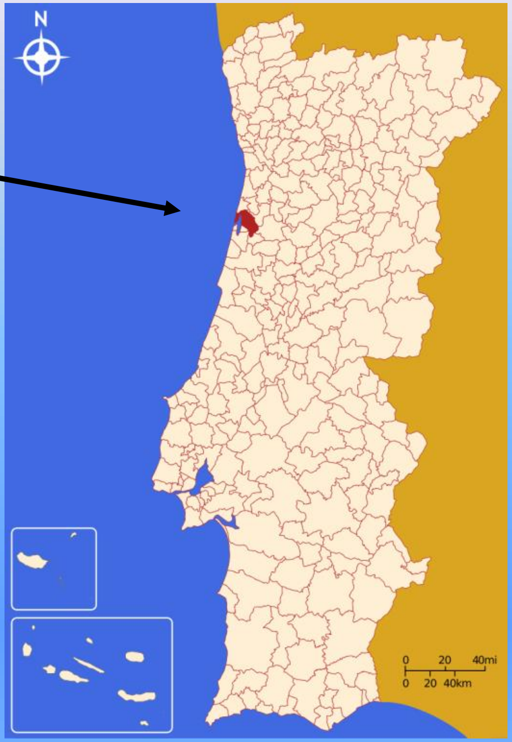
Eugen Radu
Aveiro University, Portugal (email: eugen.radu@ua.pt)

based on work done (mainly) with

C. Herdeiro

..just a few words:

Aveiro



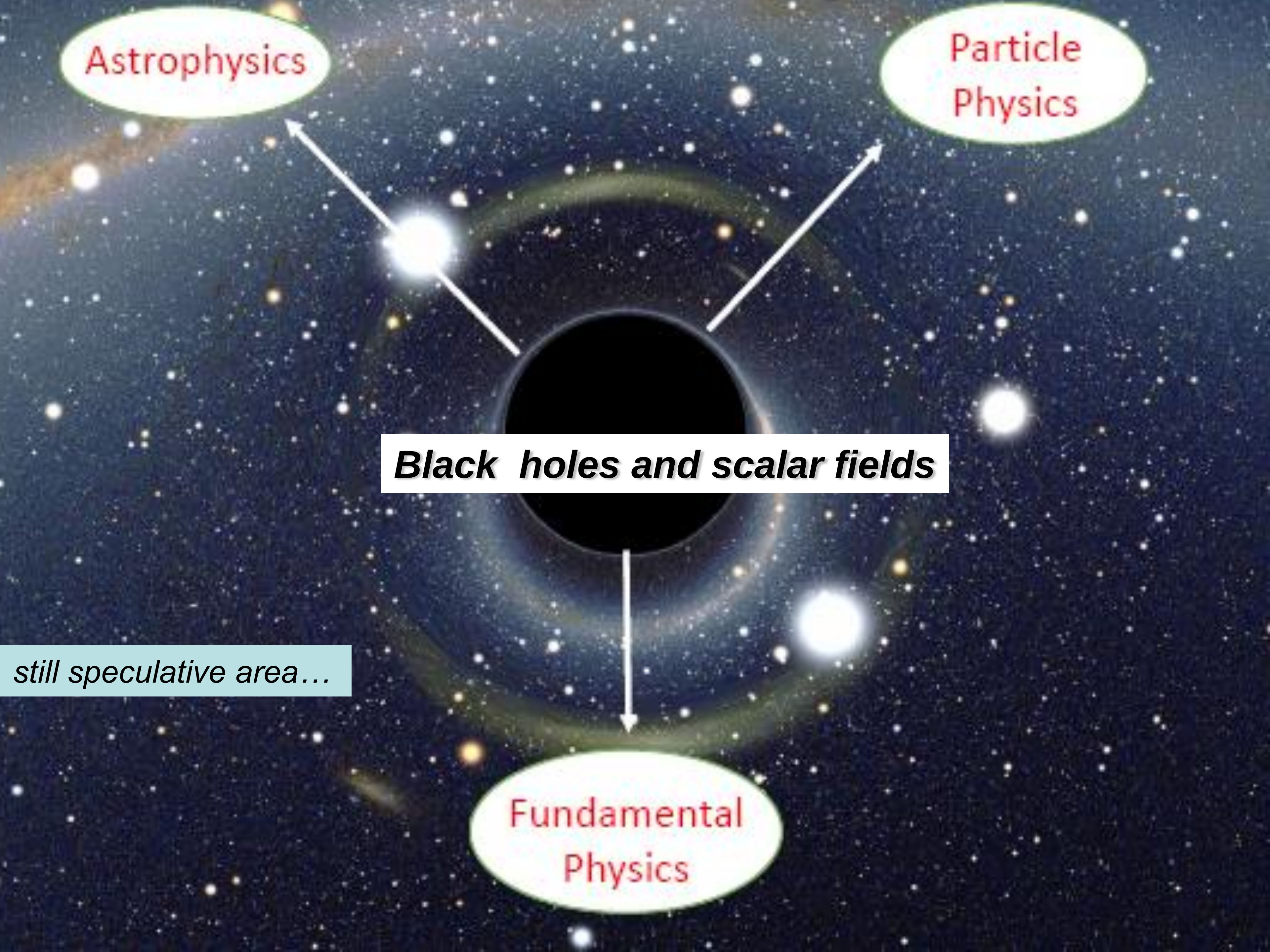
“Venetia Portugaliei”

grupul ***gr@v*** in Aveiro

research:

- general relativity
- particle physics (beyond Standard Model)
- astrophysics

Black holes and scalar fields



still speculative area…

overview:

- ***increasing evidence* for the existence of Black Holes**

- ***scalar fields:* exist in Nature (*Higgs at LHC*)**

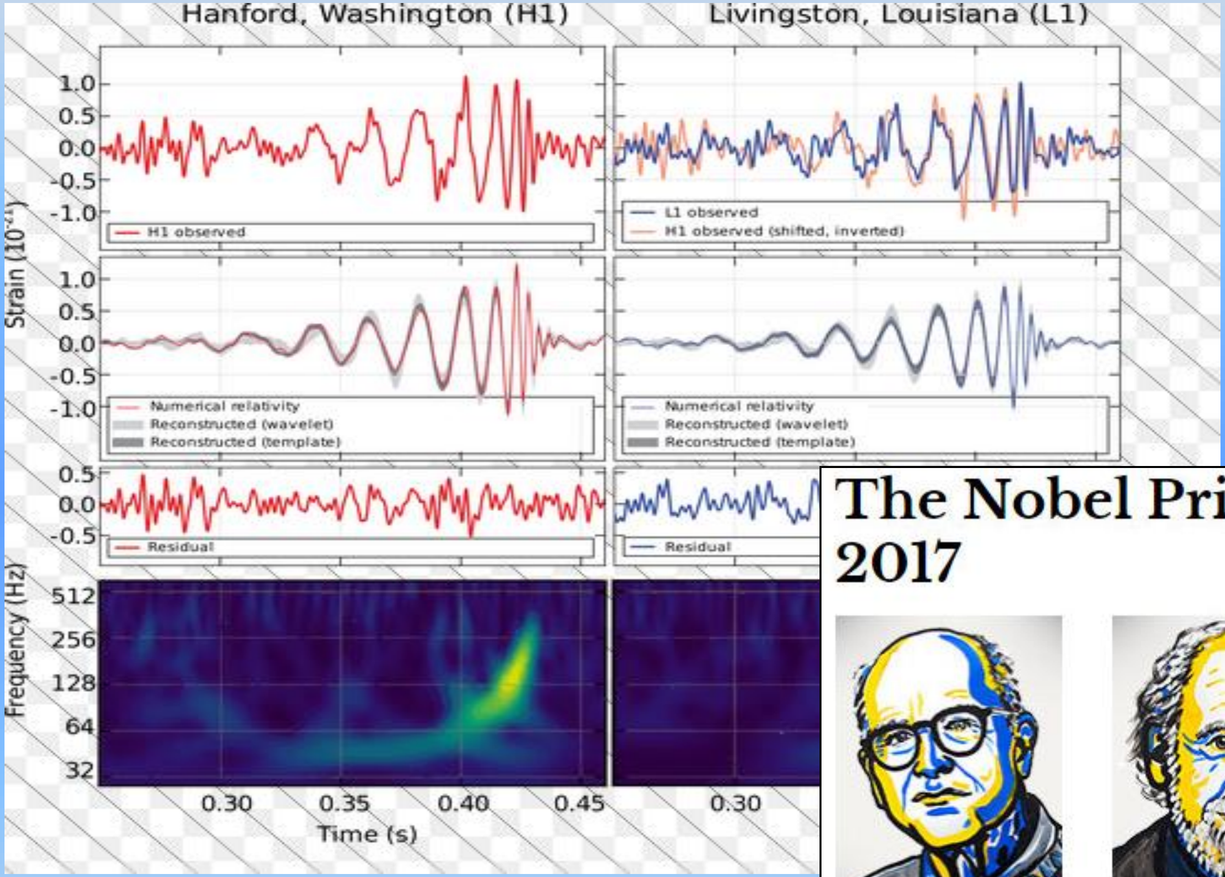
question:

- **Black Holes + scalar fields ?**

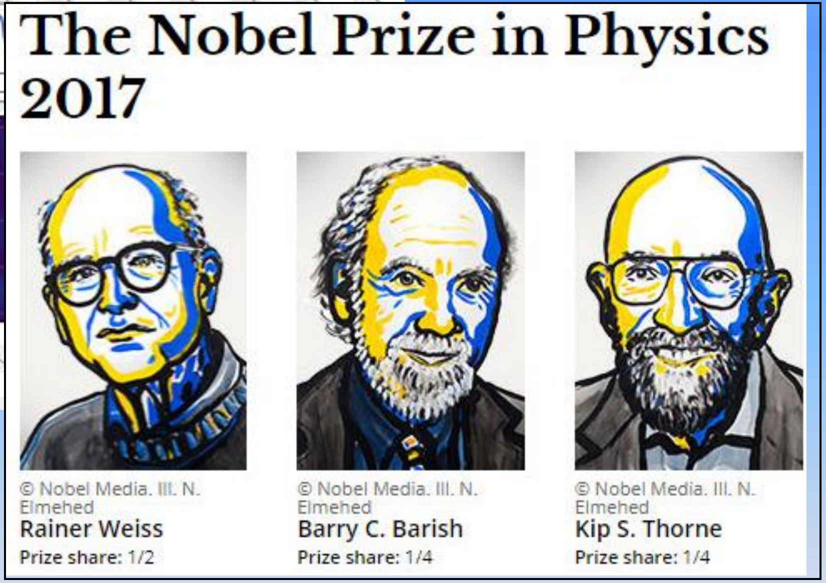
*i) **Black Holes***

great time for Black Hole physics

second Golden Age

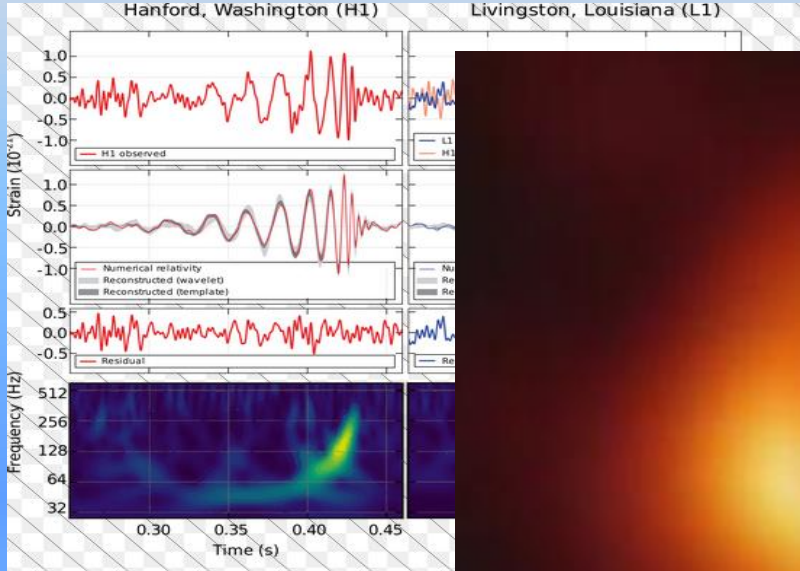


11.02.2016



great time for Black Hole physics

second Golden Age

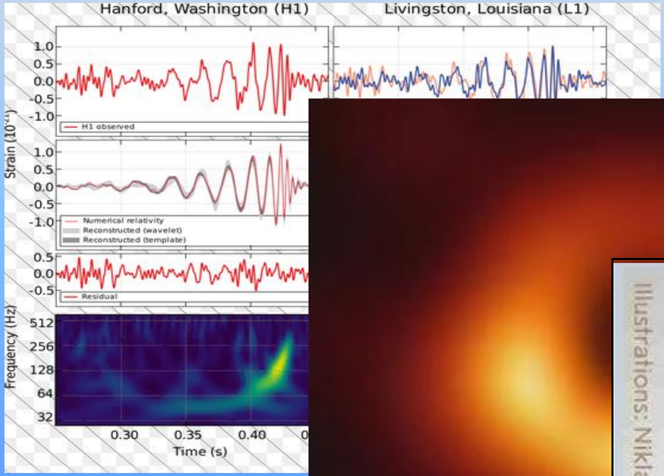


11.02.2016

10.04.2019
12.05.2022

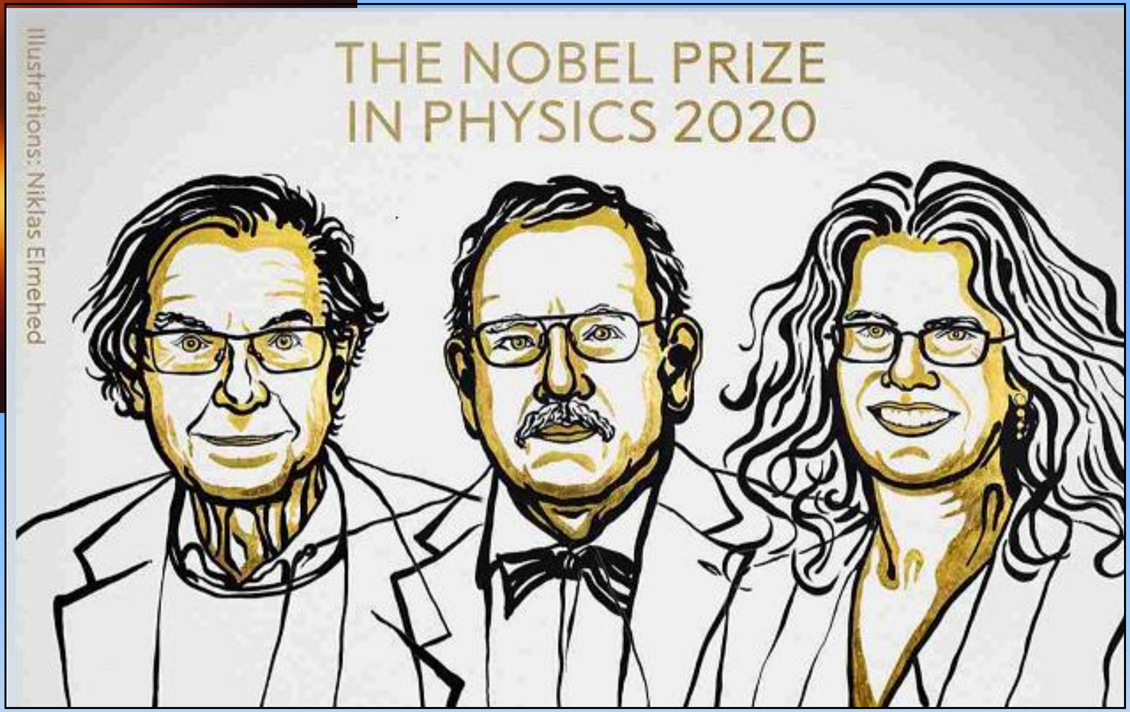
great time for Black Hole physics

second Golden Age



11.02.2016

10.04.2019
12.05.2022

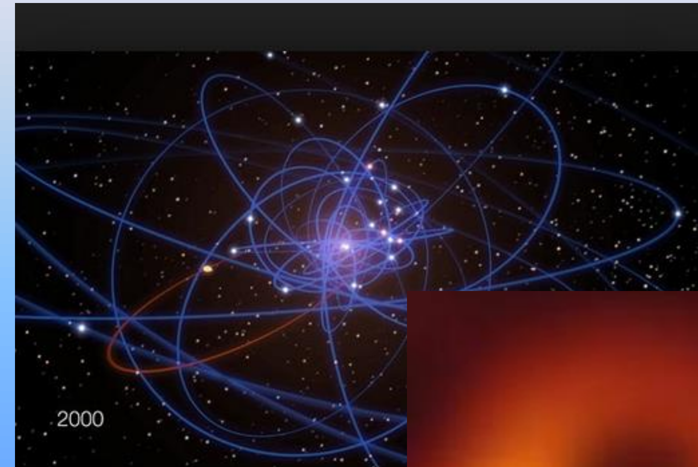


6.10.2020

(R. Penrose, R. Genzel, A. Ghez)

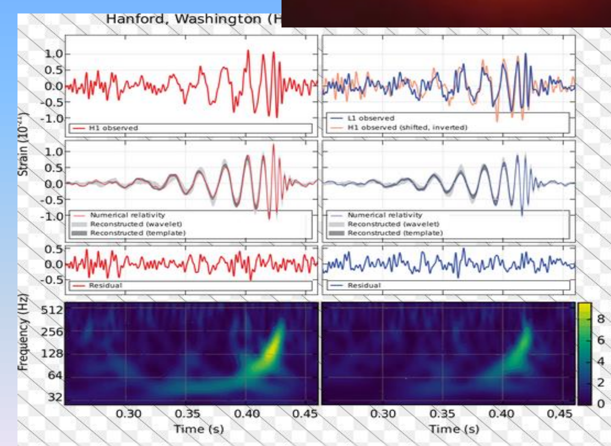
how do we know there are Black Holes?

astrophysics:



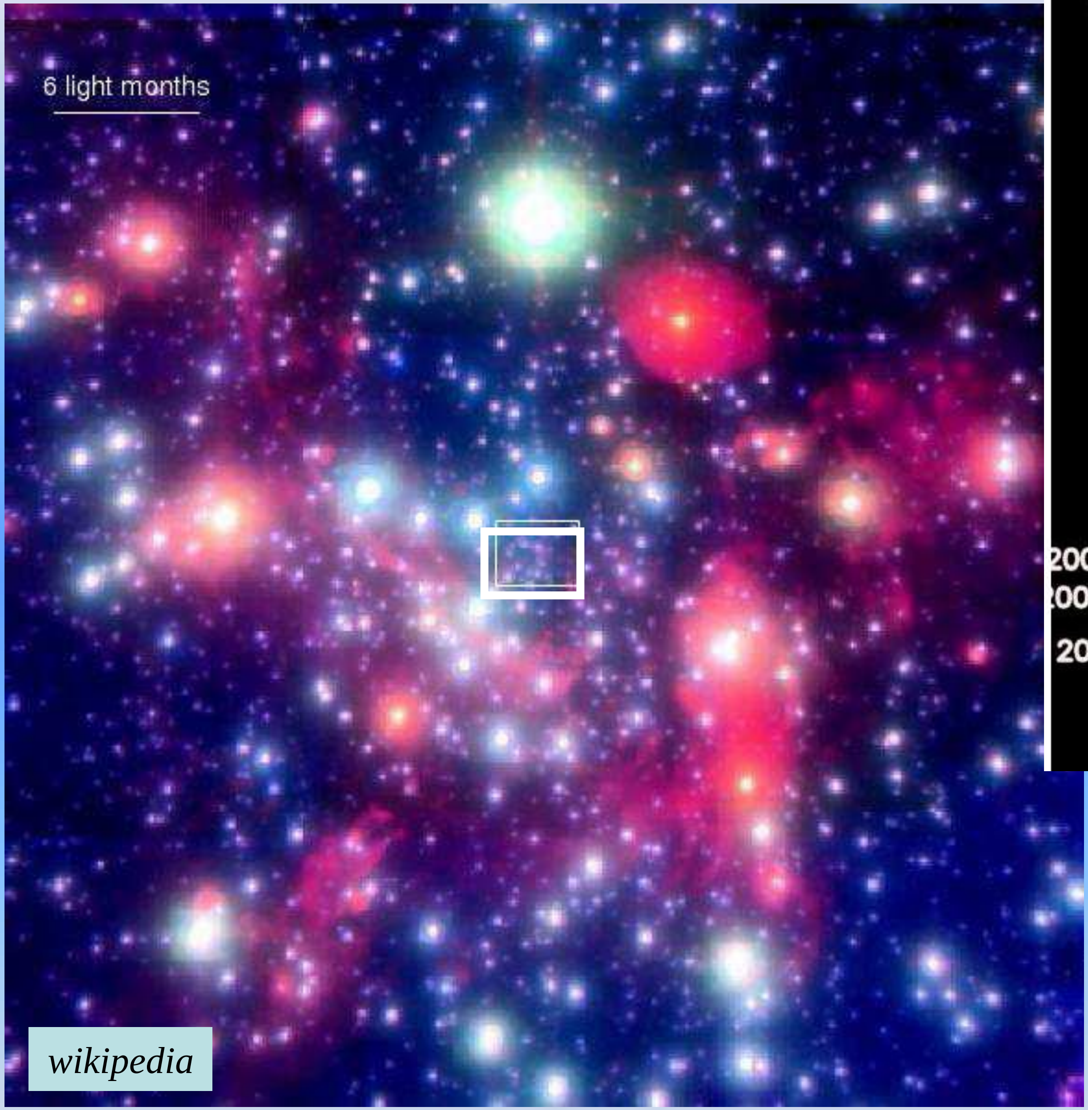
i) observation of trajectories (stars,light..)

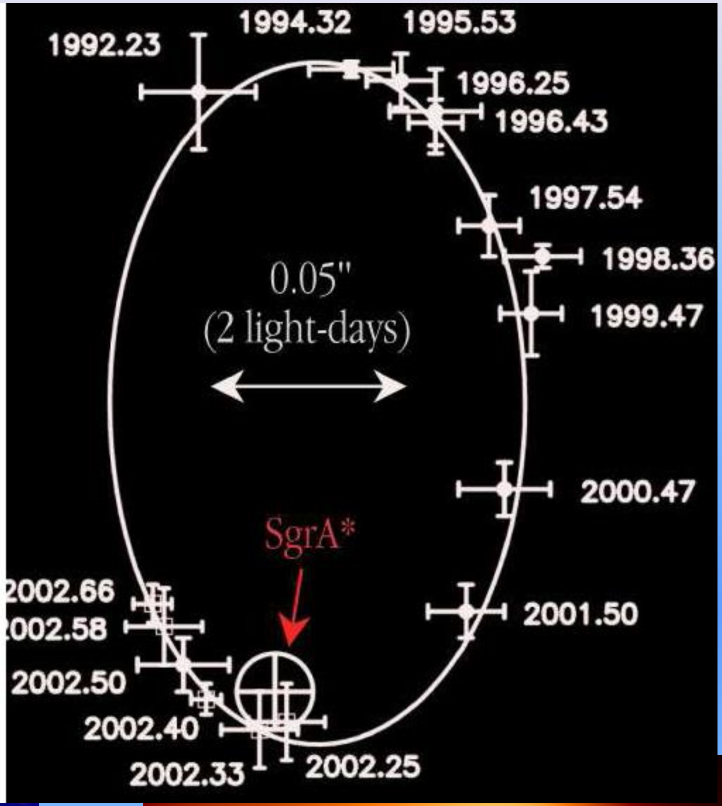
ii) detection of gravitational waves

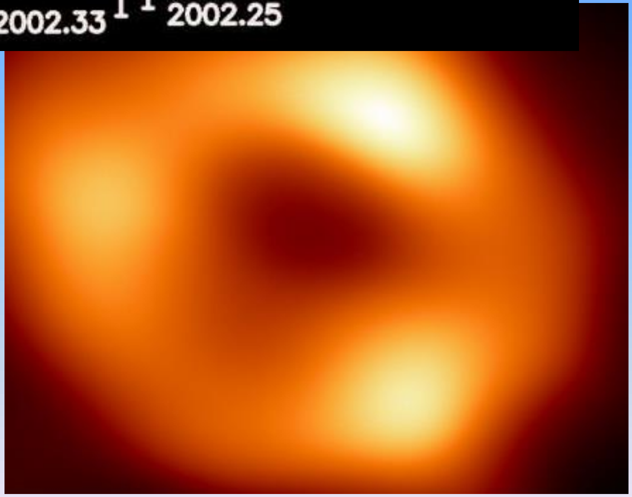


Black Holes: best explanation so far…

Black Hole at the center of the Milky Way (Sagittarius A*)



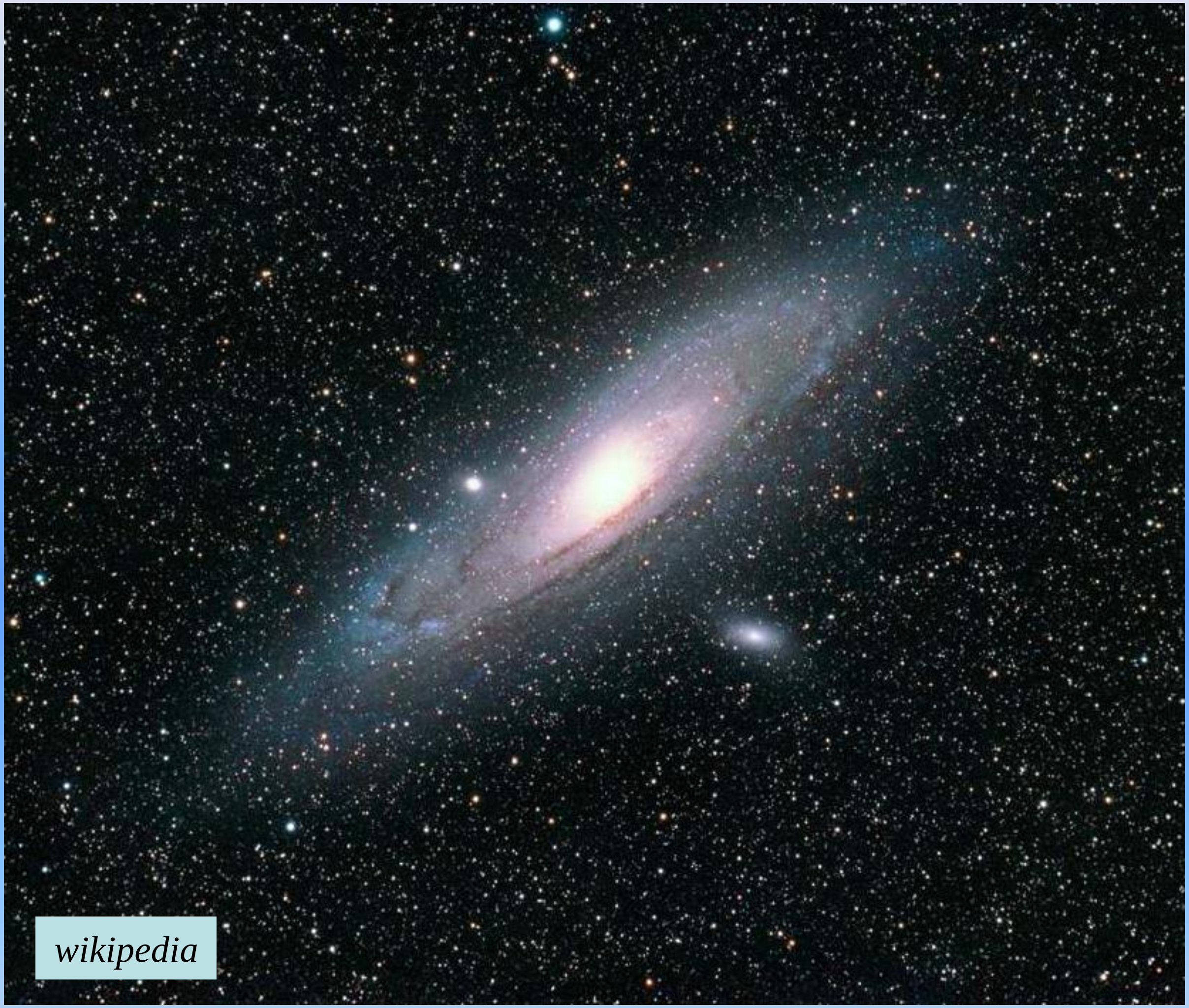


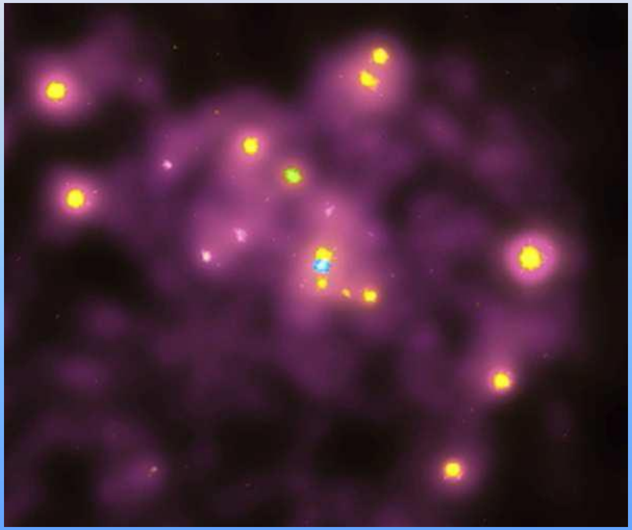


distance ~ 3x10^4 ly

mass ~ 4.1 million solar masses.

Andromeda Galaxy (=M31)

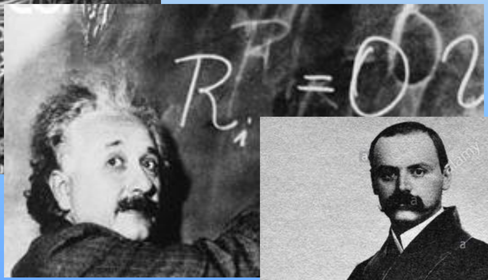


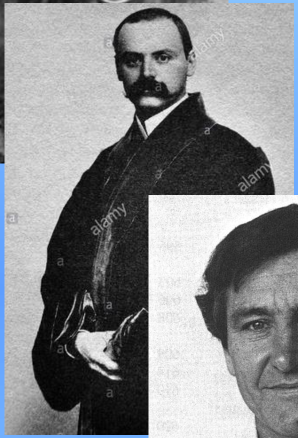


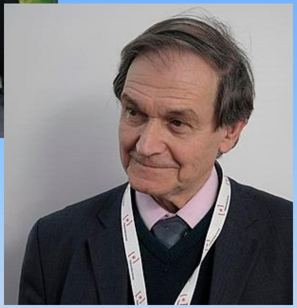
distance ~ 2.5x10^6 ly

mass ~200 million solar masses

BH overview:



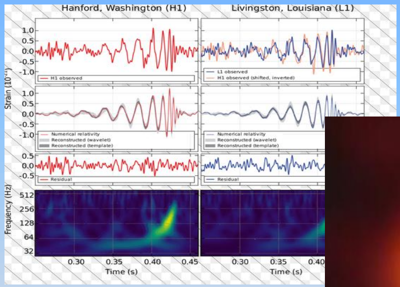




$S = k.\log W$

connection with various branches of physics

historical perspective



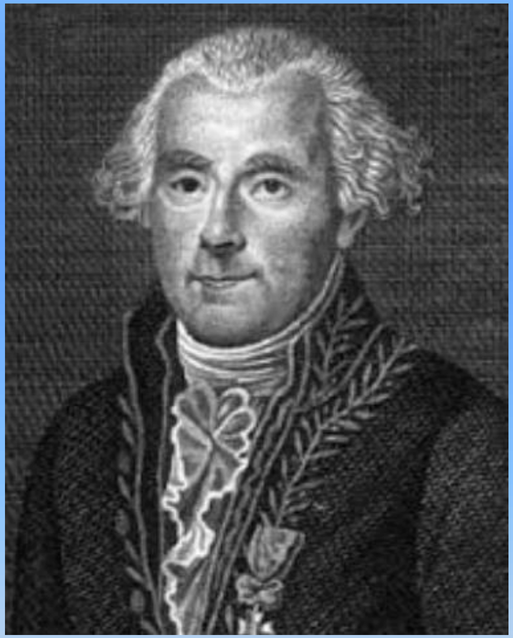
evidence for BH existence

historical perspective:

- Black Holes: ***not*** a new concept in physics:
- they were predicted already in the 18th century by *John Michell* and *Pierre-Simon Laplace*

Philosophical Transactions of the Royal Society of London (1784)

Exposition du Système du Monde (1796)



Pierre-Simon Laplace (1749-1827)

"dark stars"

several basic Black Hole properties can be understood by using *Newtonian physics*

A brief timeline

PHILOSOPHICAL

TRANSACTIONS:

On the Means of Discovering the Distance, Magnitude, &c. of the Fixed Stars, in Consequence of the Diminution of the Velocity of Their Light, in Case Such a Diminution Should be Found to Take Place in any of Them, and Such Other Data Should be Procured from Observations, as Would be Farther Necessary for That Purpose. By the Rev. John Michell, B. D. F. R. S. In a Letter to Henry Cavendish, Esq. F. R. S. and A. S.

John Michell

Phil. Trans. R. Soc. Lond. 1784 **74**, 35-57, published 1 January 1784

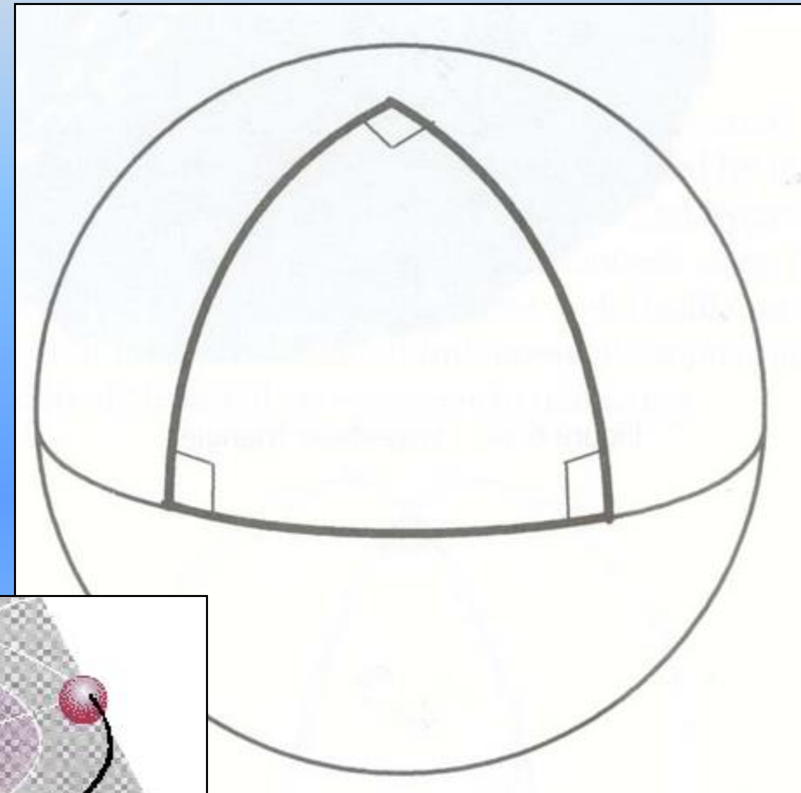
John Michell reasoned that if light was made of small particles - as Newton had proposed -these particles should feel the pull of gravity. Thus, stars with an escape velocity larger than the speed of light would be invisible for a sufficiently far away observer.

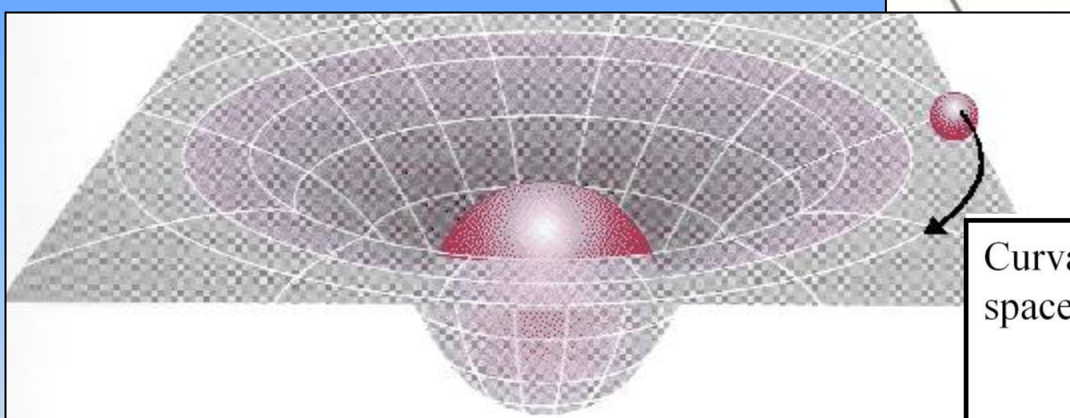
Michell: ``*if any other luminous bodies should happen to revolve about them we might still perhaps from the motions of these revolving bodies infer the existence of the central ones with some degree of probability*".

Modern description of black holes: *General Relativity*

A. Einstein (1915)

- geometric theory of gravitation: (non-Euclidean geometry)
- non-linear partial differential equations





Curvature of space

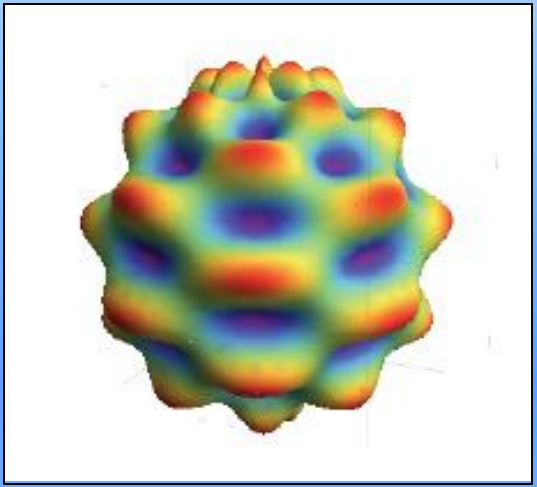
Distribution of mass/energy

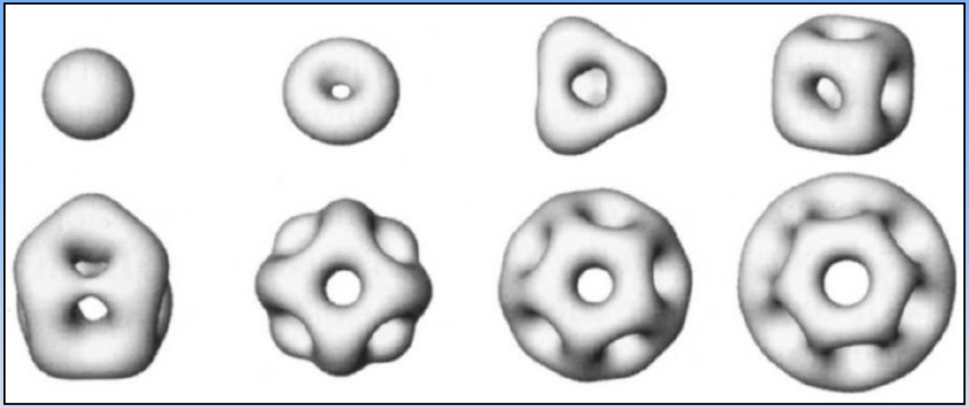
$$G_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta}$$

Some constants

Black holes are very special

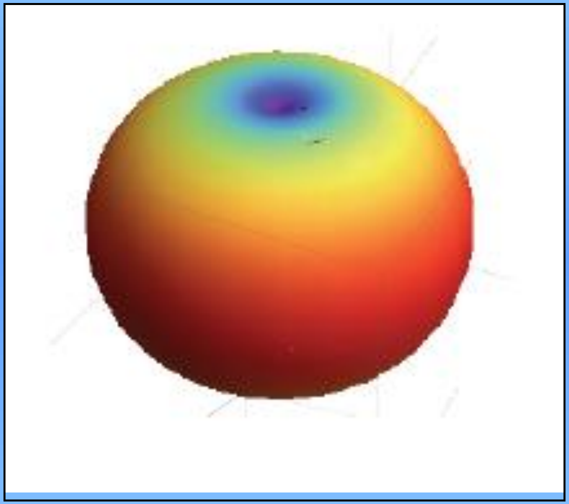
stars, solitons/field theory…





various shapes/large number of parameters

Gravity



only one solution:

Schwarzschild/Kerr Black Hole

one/two parameters

(at all scales!)

first black hole:

astronomer

the Schwarzschild solution
(1916)

$$ds^2 = \frac{dr^2}{1-\frac{2M}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2) - (1 - \frac{2M}{r})dt^2$$

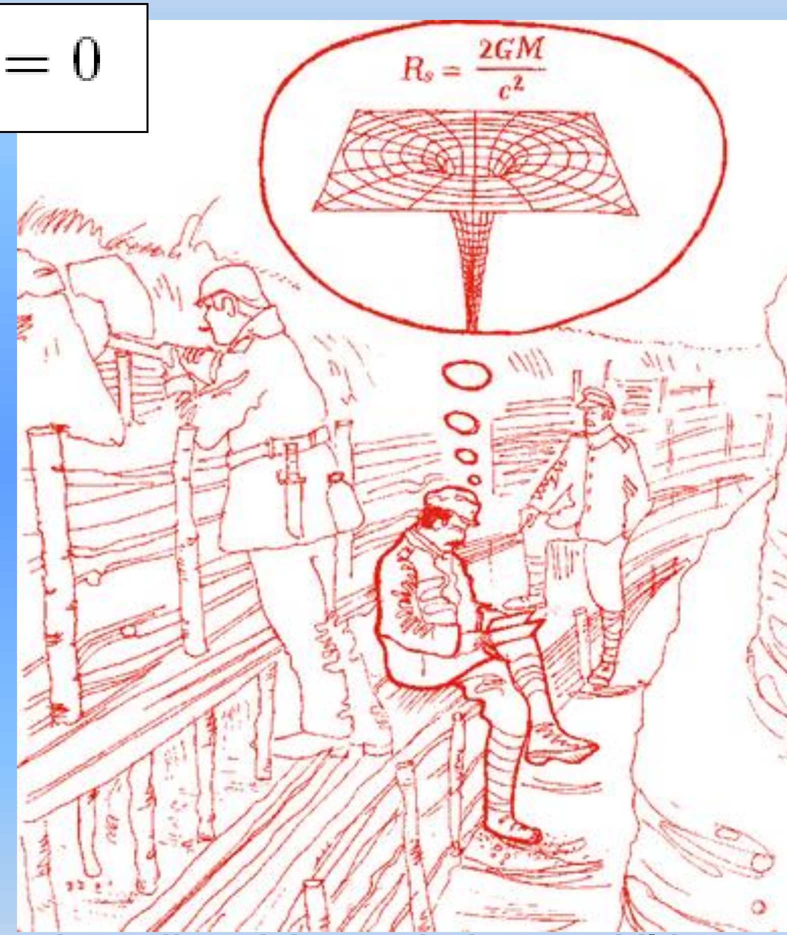
geometry:

$$G_{\alpha\beta} = 0$$

$$T_{\alpha\beta} = 0$$

no matter

- static
- vacuum
- spherically symmetric



https://spark.iop.org/schwarzschilds-war

"The war treated me kind enough, in spite of the heavy gunfire, to allow me to get away from it all and take this walk in the land of your ideas."
(letter to Einstein)

1916: **Schwarzschild's solution**

Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie.

Von K. Schwarzschild.

(Vorgelegt am 13. Januar 1916 [s. oben S. 42].)

§ 1. Hr. Einstein hat in seiner Arbeit über die Perihelbewegung des Merkur (s. Sitzungsberichte vom 18. November 1915) folgendes Problem gestellt:

Ein Punkt bewege sich gemäß der Forderung

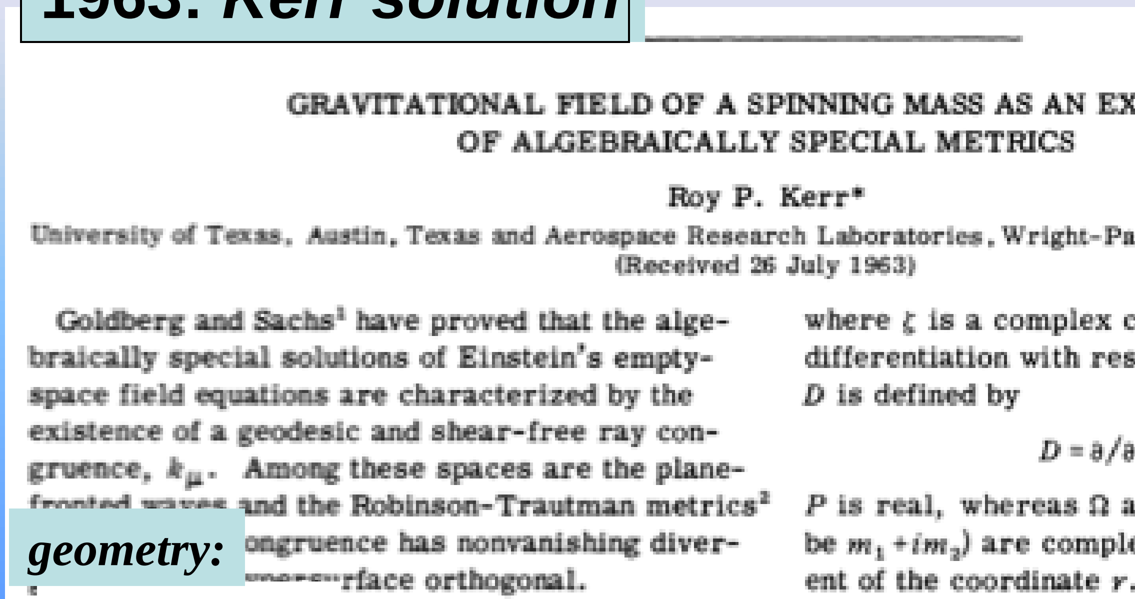
$$\delta \int ds = 0, \qquad \text{wobei} \qquad ds = \sqrt{\sum g_{\mu\nu} dx_\mu dx_\nu} \quad \mu, \nu = 1, 2, 3, 4 \tag{1}$$

ist, $g_{\mu\nu}$ Funktionen der Variabeln x bedeuten und bei der Variation am Anfang und Ende des Integrationswegs die Variablen x festzuhalten sind. Der Punkt bewege sich also, kurz gesagt, auf einer geodätischen Linie in der durch das Linienelement ds charakterisierten Mannigfaltig

$$ds^2 = \frac{dr^2}{1 - \frac{2M}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2) - (1 - \frac{2M}{r})dt^2$$

a rotating black hole

1963: *Kerr solution*



GRAVITATIONAL FIELD OF A SPINNING MASS AS AN EX
OF ALGEBRAICALLY SPECIAL METRICS

Roy P. Kerr*

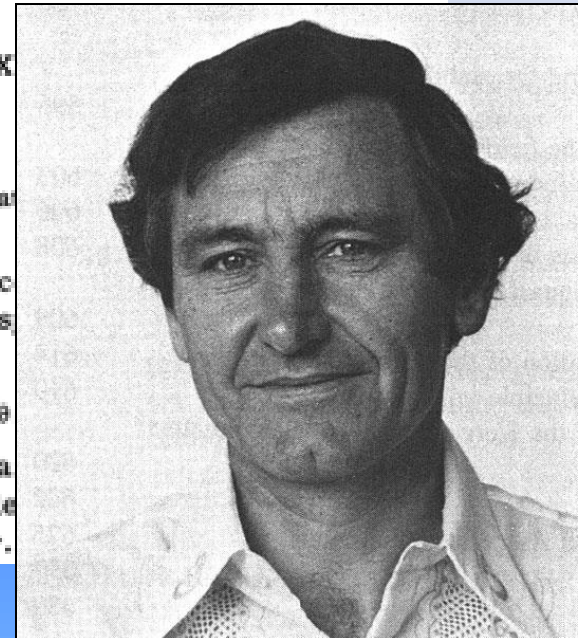
University of Texas, Austin, Texas and Aerospace Research Laboratories, Wright-Pa
(Received 26 July 1963)

Goldberg and Sachs[1] have proved that the algebraically special solutions of Einstein's empty-space field equations are characterized by the existence of a geodesic and shear-free ray congruence, k_μ. Among these spaces are the plane-fronted waves and the Robinson-Trautman metrics[2] ...ongruence has nonvanishing diver- ...rface orthogonal.

where ζ is a complex c... differentiation with res... D is defined by

D = ∂/∂...

P is real, whereas Ω a... be $m_1 + im_2$) are comple... ent of the coordinate r.



Roy Kerr

geometry:

$$G_{\alpha\beta} = 0$$

$$T_{\alpha\beta} = 0$$

no matter

the vacuum, axially symmetric, stationary Black Hole

$$ds^2 = -\frac{(\Delta - a^2\sin^2\theta)}{\Sigma}dt^2 - 2a\sin^2\theta\,\frac{(r^2+a^2-\Delta)}{\Sigma}dtd\phi + \left(\frac{(r^2+a^2)^2 - \Delta a^2\sin^2\theta}{\Sigma}\right)\sin^2\theta d\phi^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2$$

$$\Sigma = r^2 + a^2\cos^2\theta$$

$$\Delta = r^2 - 2GMr + a^2$$

a=0: spherical symmetry **(Schwarzschild solution)**

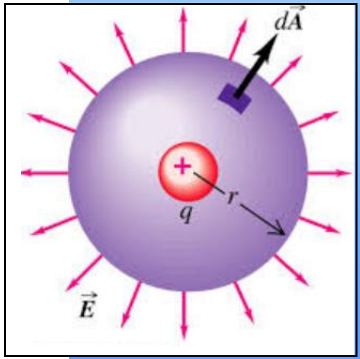
Why is the Kerr geometry so important?

UNIQUENESS OF KERR'S SOLUTION

- Kerr's solution describes *all* black holes without electric charge
- More generally,

"BLACK HOLES HAVE NO HAIR"

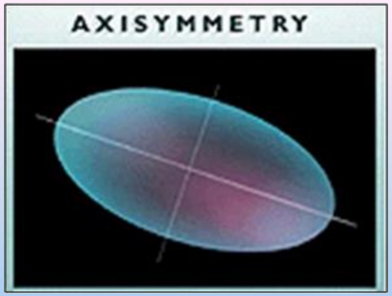
- No-hair theorem: All traces of the matter that formed a BH disappear except for:

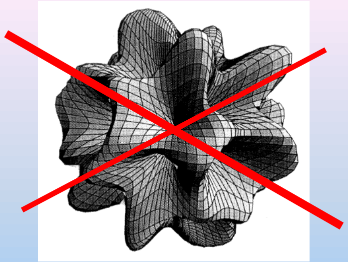


MASS
ANGULAR MOMENTUM
CHARGE

not important In astrophysics

*"**hair"** is a metaphor for any messy/ complicated details (other fields, multipoles etc)*





uniqueness theorems for Black Hole

*"In my entire scientific life, extending over forty-five years, the most shattering experience has been the realization that an exact solution of Einstein's field equations of general relativity, discovered by the New Zealand mathematician, Roy **Kerr**, provides the **absolutely exact representation** of untold numbers of black holes
that populate the Universe."*

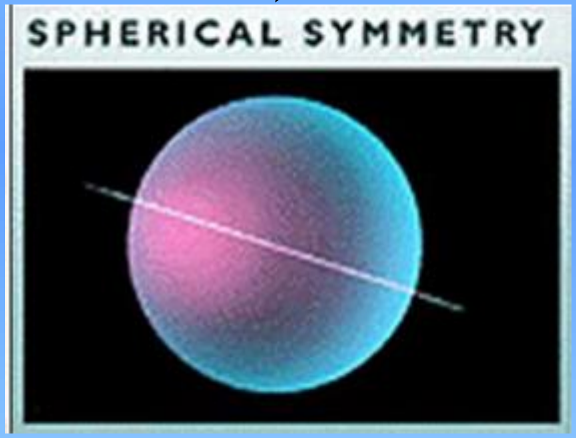
S. Chandrasekhar, in *Truth and Beauty* (1987)

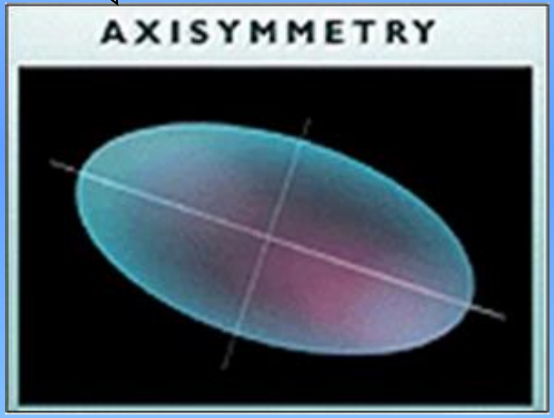
Nobel Prize (1983)

Black holes are very special

vacuum:

Schwarzschild/Kerr (unique) **solution**





- **nnly two parameters: mass *M* and angular momentum *J***
- **fully characterize the Black Hole** (at all scales)
- **very different from other cases** (e.g. stars)

*ii) **Scalar fields***

electromagnetic field *vs.* *scalar field*

Maxwell equations

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \, \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

(here, flat space)

Klein-Gordon equation

more general

monlinear-model

$$\nabla^2 \phi - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} = m^2 \phi + \lambda \phi^3$$

$$\frac{\partial V(\phi)}{\partial \phi}$$

(more general)

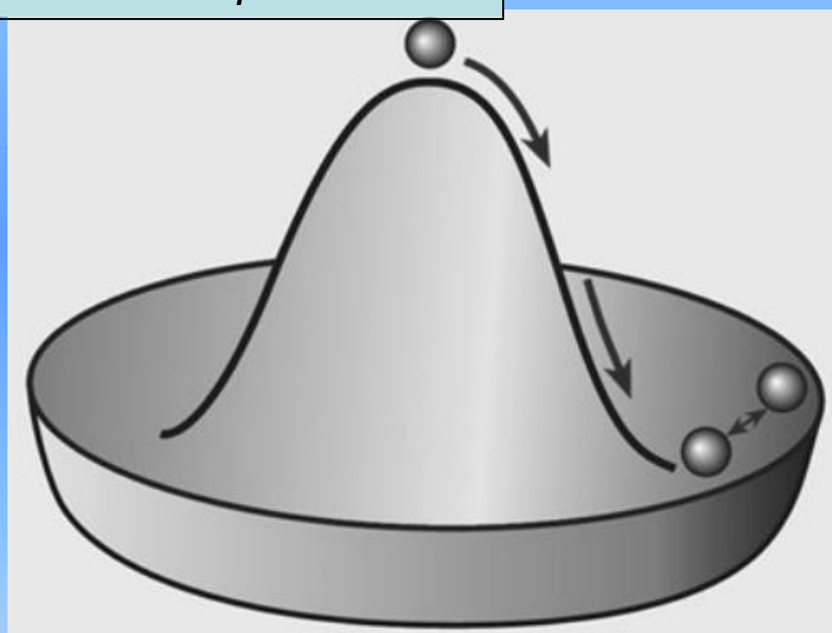
at least a scalar field exists in Nature:

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = \underbrace{m^2\phi + \lambda\phi^3}_{\frac{\partial V(\phi)}{\partial \phi}}$$

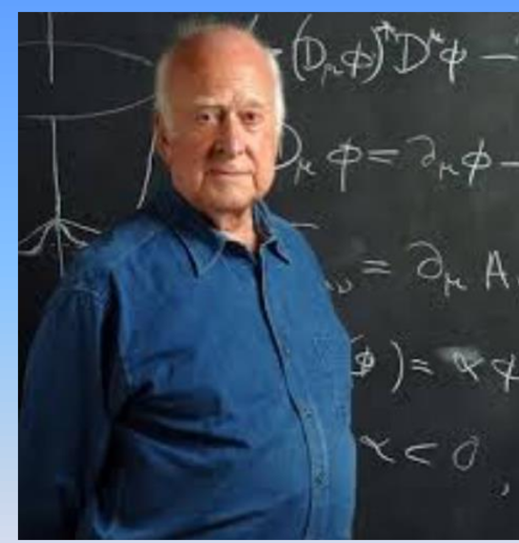
the Higgs field:

'Mexican hat' potential

special potential



- *non-linear field*
- *Interacts with other fields in the Standard Model of Particle Physics*



Peter Higgs (Nobel Prize 2013)

detected at LHC in CERN (2012)

mass ~ 125 GeV (~10^{-25} Kg)

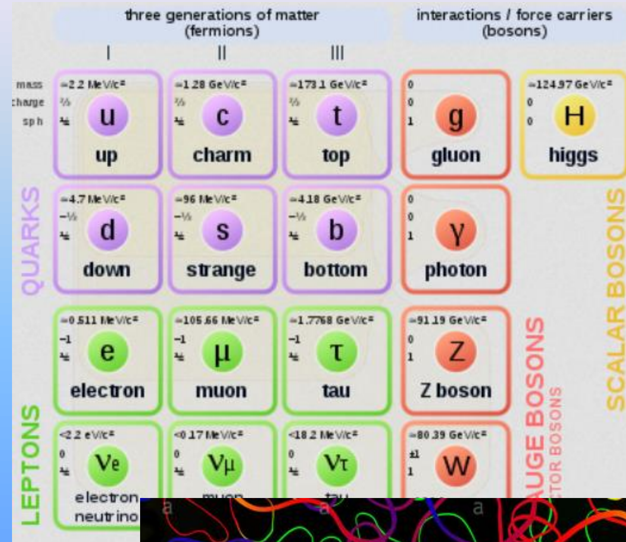
"***God particle***"

other scalar fields?

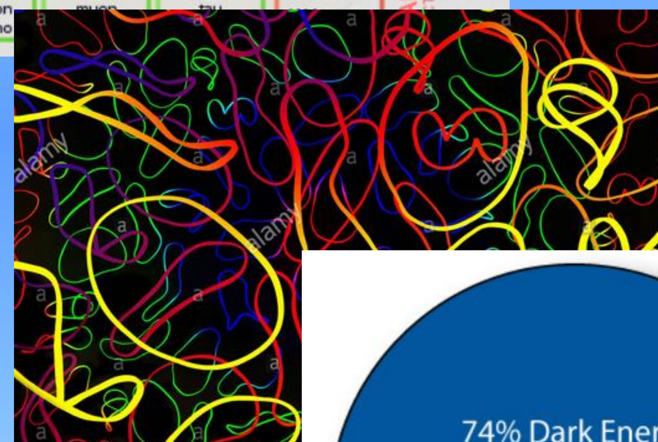
no evidence yet…

hints

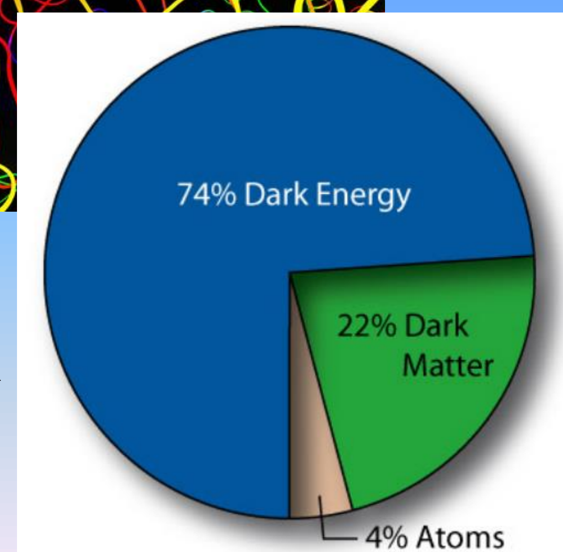
extensions of the Standard Model



String Theory +extra dims



cosmo/ogy inflation

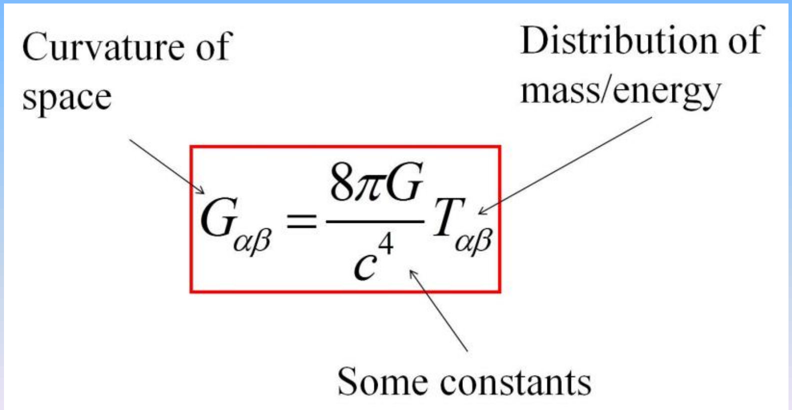


effective description..

iii) ***Black Holes***

+

Scalar Fields



the existence of Black Holes in a theory : *case by case study*

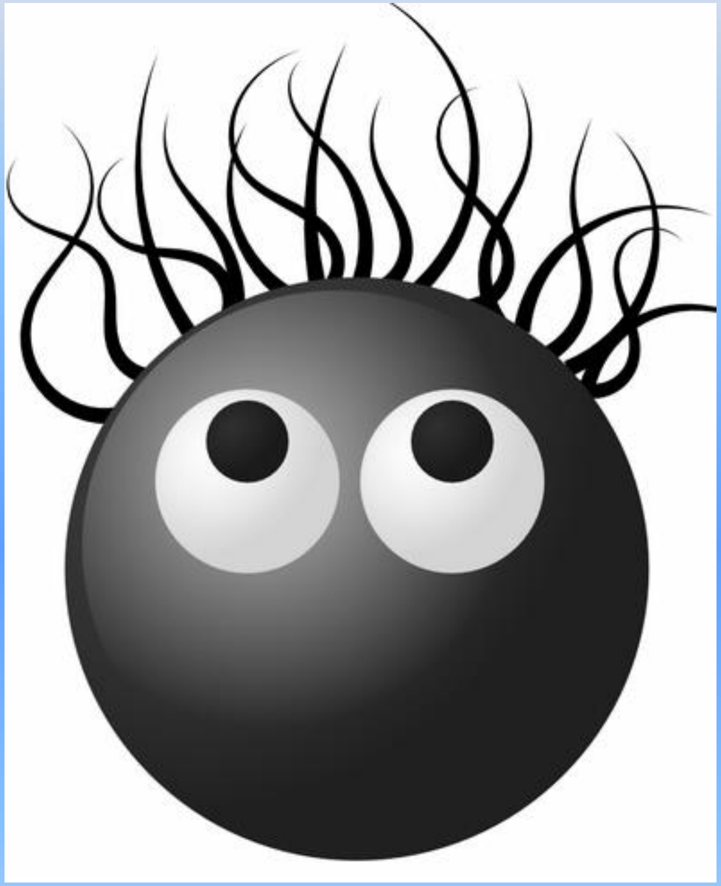
	BH Metric	Mass	Charge	Momentum
vacuum →	Schwarzschild	Yes	No	No
electromagnetic field →	Reissner–Nordström	Yes	Yes	No
vacuum →	Kerr	Yes	No	Yes
electromagnetic field →	Kerr–Newman	Yes	Yes	Yes

scalar field(s) ?

it has proven rather difficult to put together Scalar Fields and Black Holes

(and stationary configurations)

no hair theorems in Einstein-scalar field models



a Black Hole is still entirely defined by the set of parameters which are its mass, spin and electric charge, respectively

(saying the black hole has “no hair” is a metaphor for this simplicity)

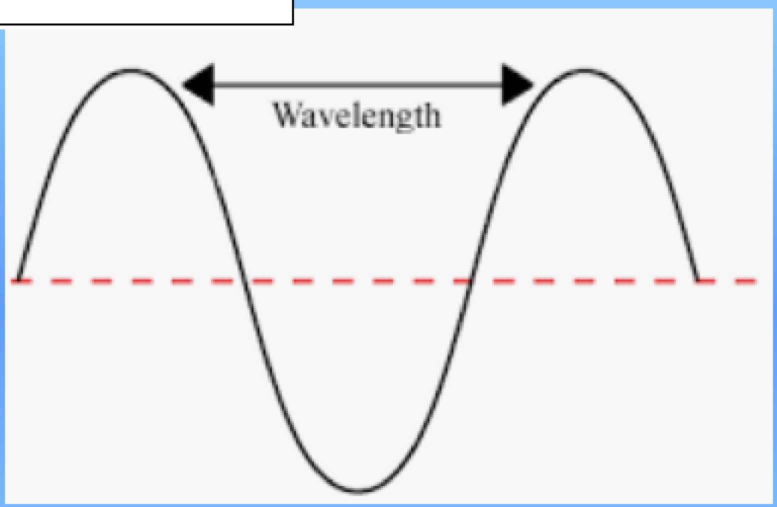
OBSERVATION:

normally (very) different characteristic scales

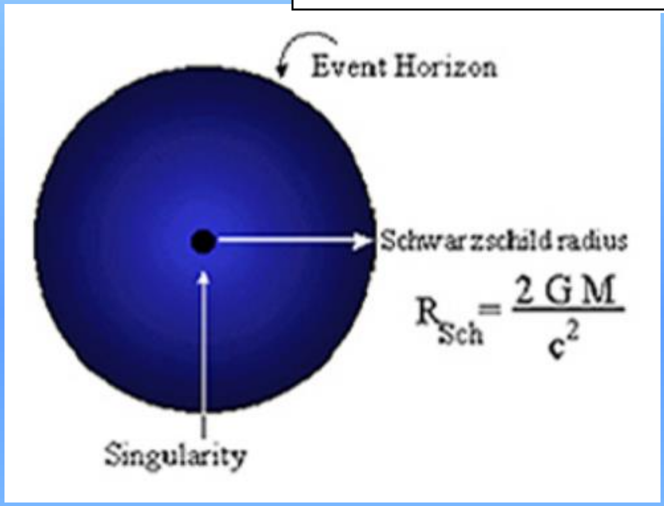
massive scalar field

gravity

$$\lambda = \frac{h}{p}$$



$$r_h = \frac{2GM}{c^2}$$



Higgs field:

$$\lambda \sim 10^{-17} m$$

Sagittarius A* Black Hole:

r_h ~ 24 million kilometers

(distance Earth-Sun: 47 million kilometers)

a (classic) no-hair theorem: (J. Bekenstein 1972)

no (static) scalar field around a Black Hole

Klein-Gordon equation

$$\nabla^2\phi = \frac{\partial V(\phi)}{\partial \phi} \quad \Longleftrightarrow \quad \phi\nabla^2\phi = \phi\frac{\partial V(\phi)}{\partial \phi}$$

Identity:

$$\phi\nabla^2\phi = \nabla(\phi\nabla\phi) - (\nabla\phi)^2$$

a no-hair theorem:

$$\nabla^2\phi = \frac{\partial V(\phi)}{\partial \phi} \quad \Longleftrightarrow \quad \phi\nabla^2\phi = \phi\frac{\partial V(\phi)}{\partial \phi}$$

Identity:

$$\phi\nabla^2\phi = \nabla(\phi\nabla\phi) - (\nabla\phi)^2$$

$$\nabla(\phi\nabla\phi) = (\nabla\phi)^2 + \phi\frac{\partial V(\phi)}{\partial \phi}$$

a no-hair theorem:

$$\nabla^2\phi = \frac{\partial V(\phi)}{\partial\phi} \iff \phi\nabla^2\phi = \phi\frac{\partial V(\phi)}{\partial\phi}$$

Identity:

$$\phi\nabla^2\phi = \nabla(\phi\nabla\phi) - (\nabla\phi)^2$$

$$\nabla(\phi\nabla\phi) - (\nabla\phi)^2 = \phi\frac{\partial V(\phi)}{\partial\phi} \iff \nabla(\phi\nabla\phi) = (\nabla\phi)^2 + \phi\frac{\partial V(\phi)}{\partial\phi}$$

a no-hair theorem:

$$\nabla^2 \phi = \frac{\partial V(\phi)}{\partial \phi} \quad \Leftrightarrow \quad \phi \nabla^2 \phi = \phi \frac{\partial V(\phi)}{\partial \phi}$$

$$\int d^3x \sqrt{-g}\, \nabla(\phi \nabla \phi) = \int d^3x \sqrt{-g} \left[(\nabla \phi)^2 + \phi \frac{\partial V(\phi)}{\partial \phi} \right]$$

$$\underbrace{\oint_\infty (\phi \nabla \phi)}_{=0} - \underbrace{\oint_H (\phi \nabla \phi)}_{=0}$$

$$\Rightarrow \int d^3x \sqrt{-g} \underbrace{\left[(\nabla \phi)^2 + \phi \frac{\partial V(\phi)}{\partial \phi} \right]}_{\geq 0} = 0$$

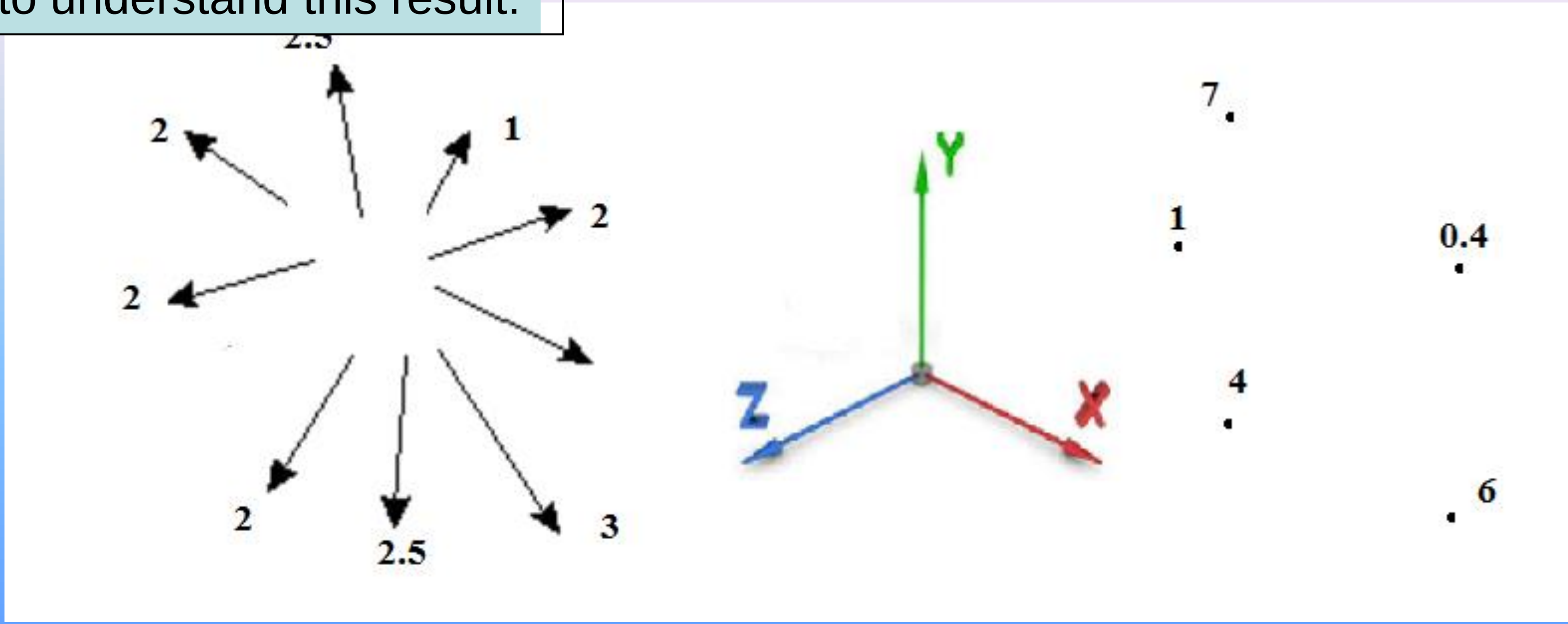
e.g. $V(\phi) = \frac{1}{2} m^2 \phi^2$

$$\int d^3x \sqrt{-g} \left[(\nabla \phi)^2 + m^2 \phi^2 \right] = 0$$

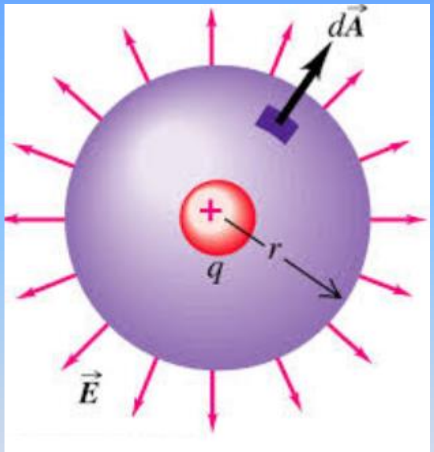
ϕ=0

Q.E.D.

how to understand this result:



electric field vs. *scalar field*

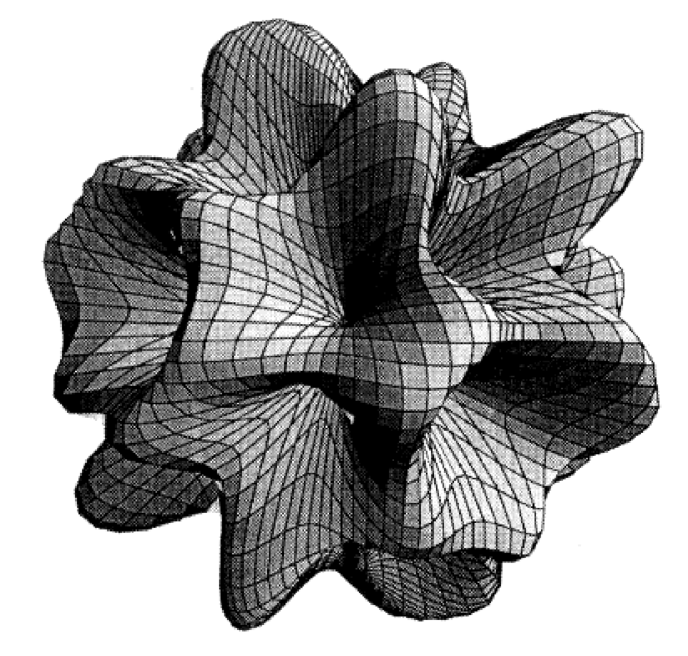


no scalar charge

non-zero flux ⟹ global (electric) charge

LOOPHOLES?

black holes with 'scalar hair'?



Yes – several different mechanisms

- *various recent developments*
- *active field of research*

Herdeiro and Radu: *the scalar no-hair theorems have a loophole*

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Black Holes with scalar fields

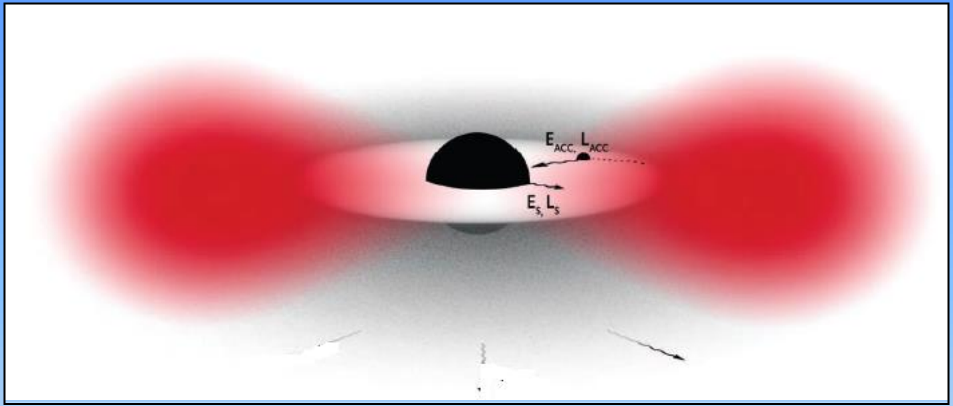
(simplest example of ‘hairy’ black holes)

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R - \Phi^*_{,a}\Phi^{,a} - \mu^2\Phi^*\Phi\right]$$

vacuum Kerr is a solution

numerics

existence proof by mathematicians



regular, stationary, asymptotically flat black holes
<u>with scalar hair</u>

spin-one

naively, such solutions should be simpler than Kerr-Newman

<u>however:</u>

standard assumption

richer, different pattern from Kerr

spin-one

naively, such solutions should be simpler than Kerr-Newman:

<u>however:</u>

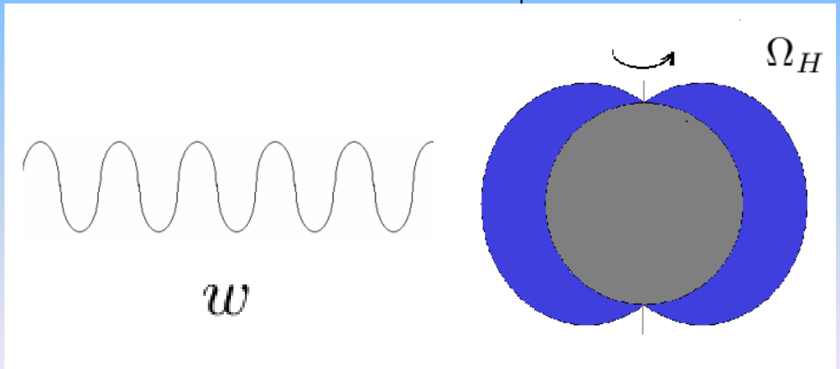
different pattern from Kerr

<u>synchronization condition</u>

: (circumvent no-hair theorems)

- **no static limit** $w = m\Omega_H$

with $\Phi \sim e^{i(m\varphi - wt)}$



general properties:

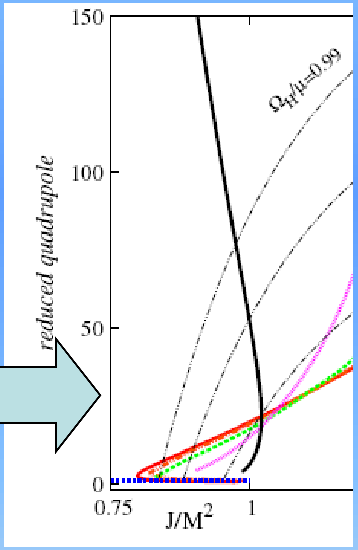
different pattern from Kerr

- **no static limit**
- **can violate Kerr bound** $J/M^2 > 1$

general properties:

different pattern from Kerr

- **no static limit**
- **violate Kerr bound**
- **different quadrupole**



general properties:

different pattern from Kerr

- **no static limit**
- **violate Kerr bound**
- **different quadrupole**
- **distinct orbits (ISCOs)**

general properties:

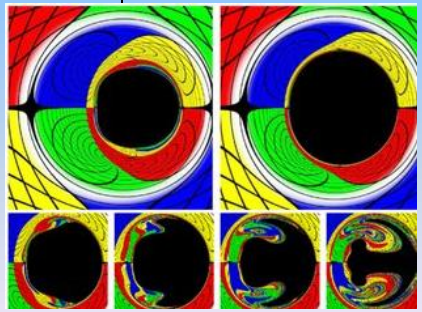
different pattern from Kerr

- **no static limit**
- **violate Kerr bound**
- **different quadrupole**
- **distinct orbits**
- **ergo-Saturns**

general properties:

different pattern from Kerr

- **no static limit**
- **violate Kerr bound**
- **different quadrupole**
- **distinct ISCOs**
- **ergo-Saturns**
- **different shadows**

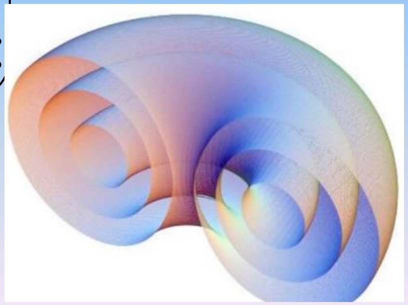


general properties:

different pattern from Kerr

- **no static limit**
- **violate Kerr bound**
- **different quadrupole**
- **distinct ISCOs**
- **ergo-Saturns**
- **solitonic limit** (no BH):

Boson Stars



the double Black Hole system with a scalar field

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R - \Phi^*_{,a}\Phi^{,a} - \mu^2\Phi^*\Phi\right]$$

not possible in (electro-)vacuum

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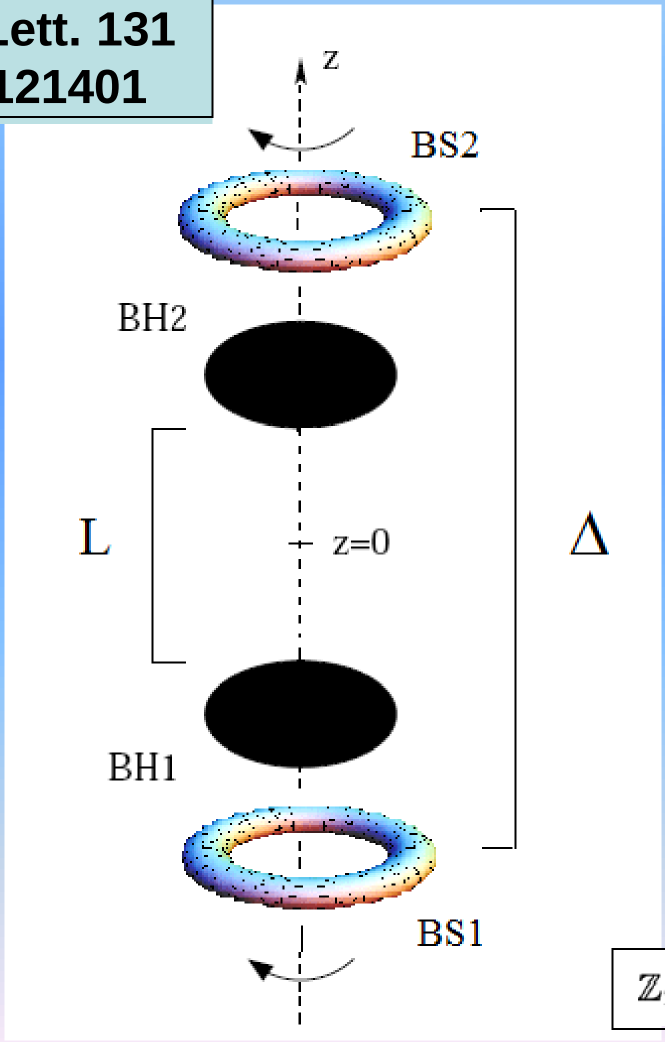


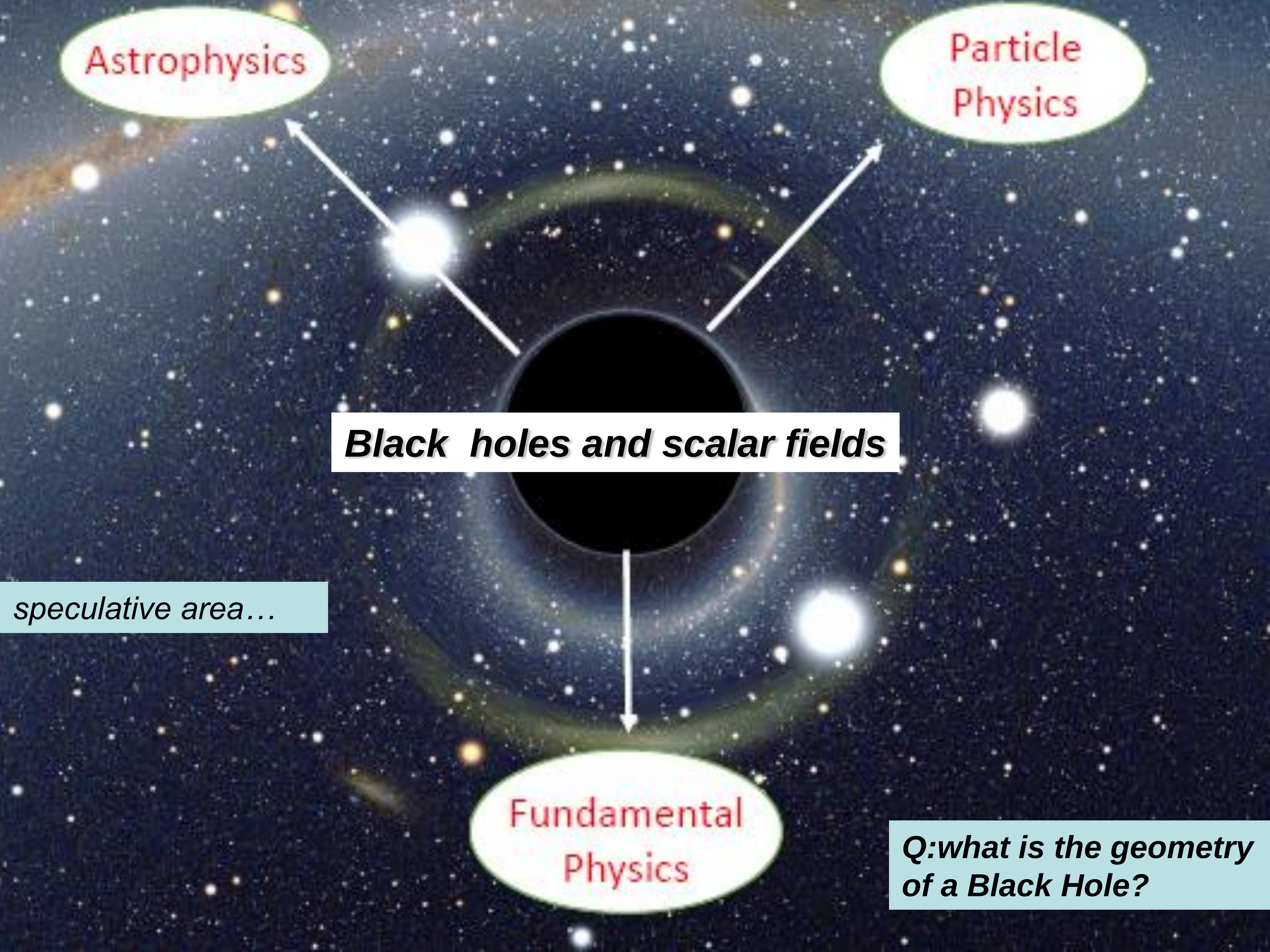
- no singularities
  (also conical)
- crucial ingredient:
  the dipolar BSs limit
- likely holds for other systems with two solitons

synchronization condition

$$w = m\Omega_H$$

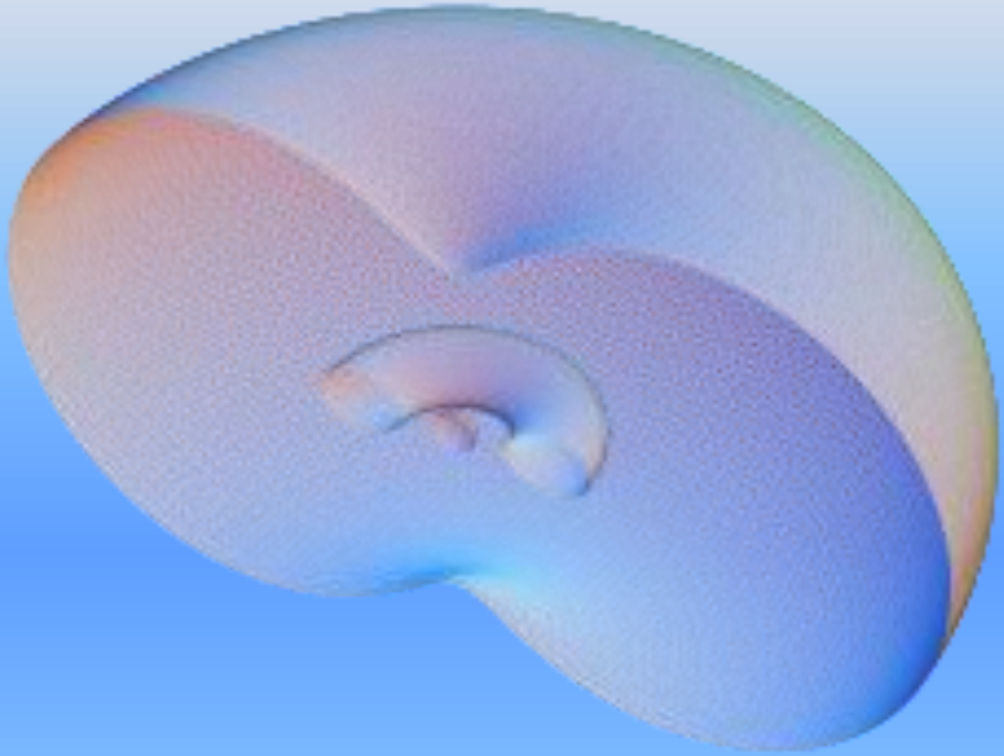
$\mathbb{Z}_2$*-symmetric solutions*

Black holes and scalar fields



speculative area…

Q:what is the geometry of a Black Hole?



Vă mulțumesc pentru atenție!