Partial Banana Mapping: impact probability estimation and precovery of asteroids

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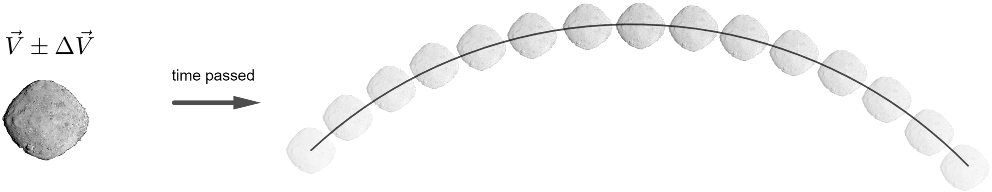
PEGASE team, IMCCE, Observatoire de Paris

1. Impact probability estimation

Asteroid's position uncertainty



Uncertainty along the orbit

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Analogy

The Monte Carlo method

From the cloud of Virtual Asteroids (VAs) at the epoch of observations we randomly choose VAs according to the distribution function. The impact probability P_{MC} is:

$$P_{MC} = \frac{k}{n}$$

k is a number of VAs, which collide with the Earth,
n is a number of all considered VAs.
The mean square error δ_{MC}:

$$\delta_{MC} = \frac{\sqrt{P_{MC}(1 - P_{MC})}}{\sqrt{n}} \approx \frac{1}{\sqrt{k}} * 100\%$$

Disadvantages:
Time consuming.
Impractical for searching for possible impactors.

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Linear methods

- A linear relation between errors of orbital parameters at different epochs is assumed
- Consequently errors of orbital parameters have a Gaussian distribution at all considered times
- Uncertainty region is constructed from a covariance matrix at each time
- The covariance matrix is computed by propagation of the nominal orbit with variational equations
- Extremely important the choice of orbital parameters (coordinates and velocities / Keplerian elements ...)

Advantages: Fast

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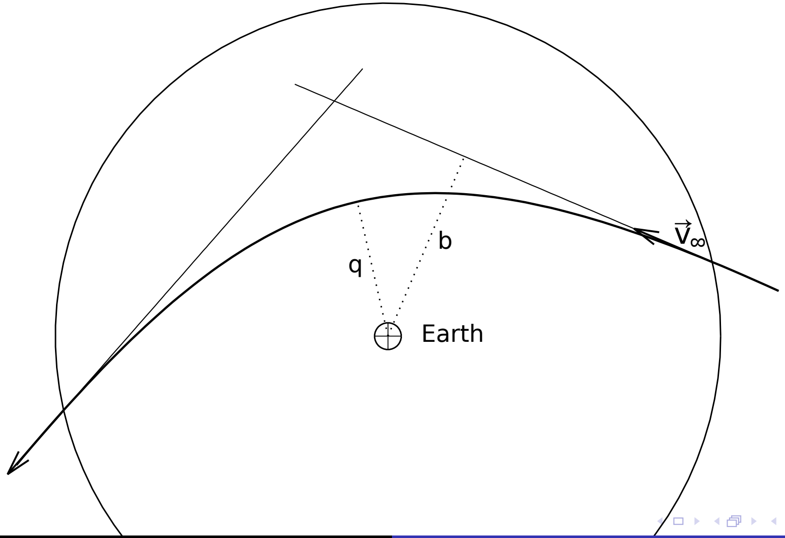
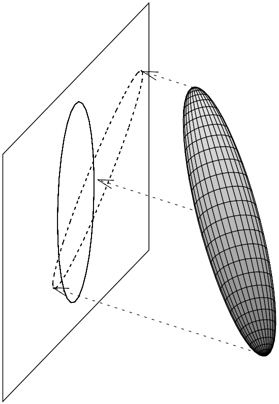
Target plane method

Integrate the asteroid's orbit until it enters the Earth's sphere of action ($\approx$ 929,000 km)

Target plane — plane perpendicular to incoming asymptote of the hyperbola ($\vec{v}_\infty$)



Target plane method



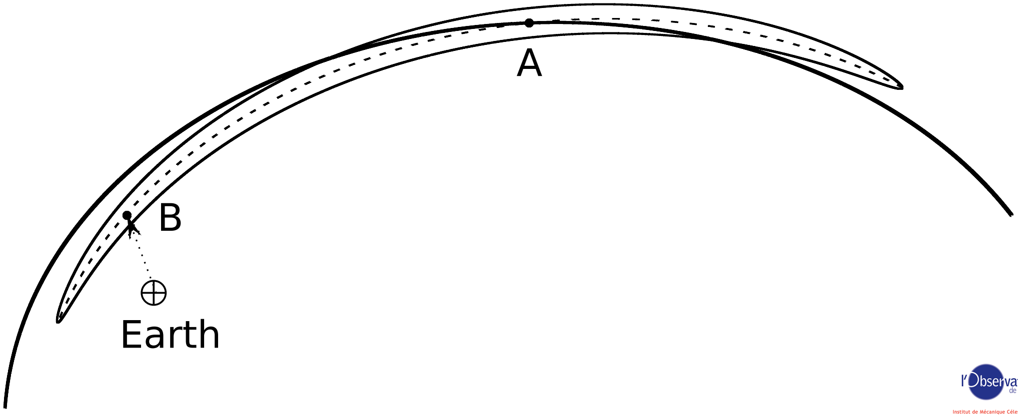
1. Integrate the asteroid's
orbit until it enters the Earth's sphere of influence.
2. The uncertainty region is represented as an ellipsoid
3. Map the uncertainty ellipsoid onto the target plane
4. Impact probability

$$P = \frac{1}{2\pi|\mathrm{det}\mathbf{L}|^{\frac{1}{2}}} \int_{S_{R'_\oplus}} e^{-(\boldsymbol{u}^{\mathrm{T}}\mathbf{L}^{-1}\boldsymbol{u})/2} \mathrm{d}\boldsymbol{u}$$

where $S_{R'_\oplus}$ is the projection
of the Earth (circle) with radius $R'_\oplus = R_\oplus\sqrt{1 + \frac{v_s^2}{v_\infty^2}}$.
v_s is the escape velocity (≈ 11.186 km/s)

Problems

- The uncertainty region is mostly stretched along the nominal asteroid's orbit
- The region is very thin ($\approx$ several times smaller than the radius of the Earth)
- But it is very long (can be the whole orbit and even more)
- If the 3-σ uncertainty is the whole orbit the impact probability value can be up to $\approx 10^{-4}$



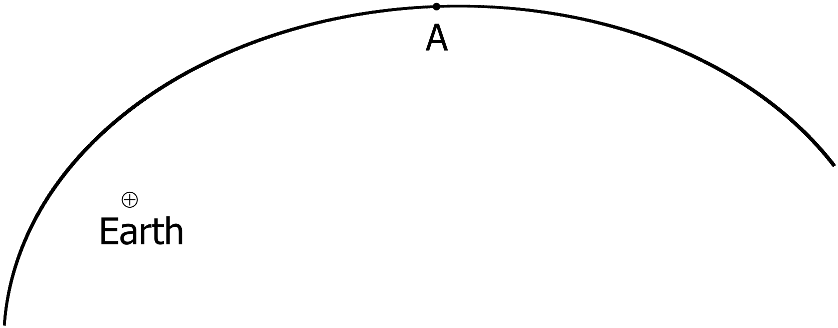
Partial Banana Mapping method: scheme

Integrate asteroid's orbit until the Earth is close to the osculating orbit.

Construct a banana-shaped uncertainty region.

Find closest to the Earth point B along main line of the banana and compute probability for B by target plane method.

Compute how to change the original orbit so that asteroid goes to point B and collide.



A — nominal position of an asteroid at a possible collision

B — probable position of an asteroid which is closest to the Earth

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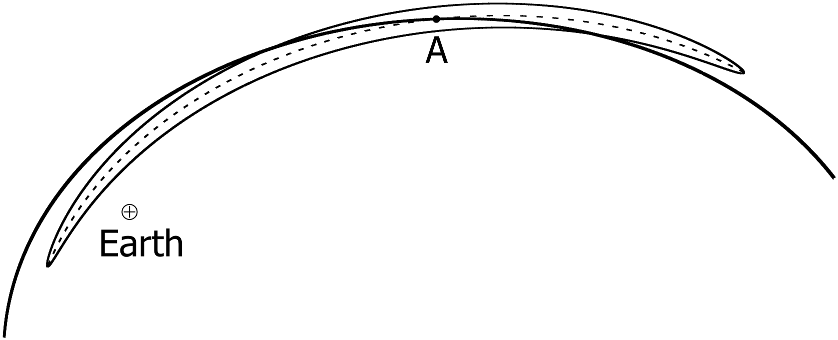

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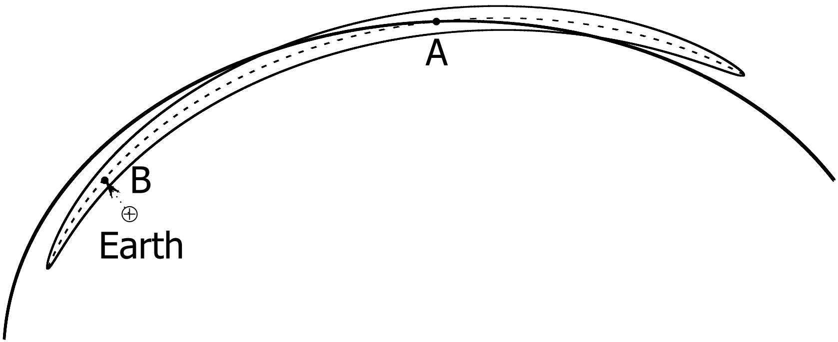

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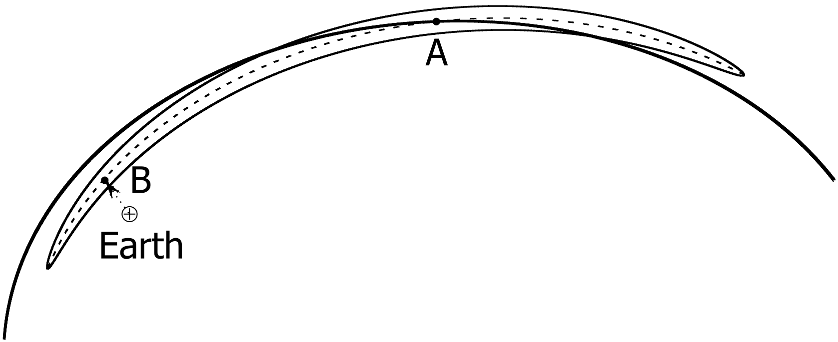

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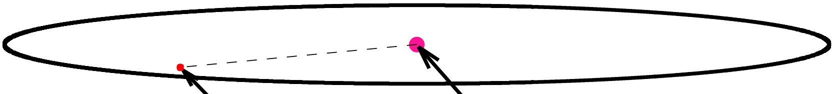
Finding collisional orbit



x_0 — state vector at epoch of observations
x — state vector at time of possible collision
E — Keplerian elements at time of possible collision
E^{min} — Keplerian elements of point B

$$\mathrm{Q} = \left[\frac{\partial E}{\partial x}\right]\left[\frac{\partial x}{\partial x_0}\right]$$

$$\mathrm{Q}^{-1}[E^{min} - E] = [x_0^{min} - x_0]$$

Use x_0^{min}, integrate this orbit and compute impact probability
Multiply probability by a factor $\exp(-\frac{1}{2}[x_0^{min} - x_0]^T C^{-1}[x_0^{min} - x_0])$

Comparison of impact probabilities

Designation	$P_{MC} \pm 3\sigma_{MC}$	P_{TP}	P_{PBM}	P_{IPBM}
2006 JY26	$(5.6 \pm 1.7) \cdot 10^{-5}$	$\mathbf{1.0 \cdot 10^{-4}}$	$\mathbf{1.0 \cdot 10^{-4}}$	$6.0 \cdot 10^{-5}$
2006 QV89	$(1.8 \pm 0.1) \cdot 10^{-3}$	$\mathbf{2.0 \cdot 10^{-3}}$	$1.8 \cdot 10^{-3}$	$1.8 \cdot 10^{-3}$
2010 UK	$(3.1 \pm 0.7) \cdot 10^{-3}$	$\mathbf{1.8 \cdot 10^{-3}}$	$\mathbf{1.8 \cdot 10^{-3}}$	$2.9 \cdot 10^{-3}$
2011 AG5	$(5.3 \pm 1.3) \cdot 10^{-4}$	$4.2 \cdot 10^{-4}$	$5.7 \cdot 10^{-4}$	$5.5 \cdot 10^{-4}$
2007 VK184	$(6.2 \pm 2.0) \cdot 10^{-6}$	$\mathbf{2.7 \cdot 10^{-5}}$	$\mathbf{2.6 \cdot 10^{-5}}$	$8.8 \cdot 10^{-6}$
2007 VE191	$(6.4 \pm 1.0) \cdot 10^{-4}$	$\mathbf{0.0}$	$6.8 \cdot 10^{-4}$	$7.0 \cdot 10^{-4}$
2008 JL3	$(3.0 \pm 0.4) \cdot 10^{-4}$	$\mathbf{7.5 \cdot 10^{-4}}$	$\mathbf{4.0 \cdot 10^{-4}}$	$3.3 \cdot 10^{-4}$
2014 WA	$(3.5 \pm 2.4) \cdot 10^{-7}$	$\mathbf{0.0}$	$5.4 \cdot 10^{-7}$	$5.3 \cdot 10^{-7}$
2009 JF1	$(7.4 \pm 1.2) \cdot 10^{-4}$	$7.3 \cdot 10^{-4}$	$8.0 \cdot 10^{-4}$	$7.6 \cdot 10^{-4}$
2012 MF7	$(3.1 \pm 0.8) \cdot 10^{-4}$	$\mathbf{0.0}$	$\mathbf{4.8 \cdot 10^{-4}}$	$3.2 \cdot 10^{-4}$
2008 CK70	$(6.4 \pm 1.0) \cdot 10^{-4}$	$5.8 \cdot 10^{-4}$	$6.9 \cdot 10^{-4}$	$6.5 \cdot 10^{-4}$
2005 BS1	$(1.4 \pm 0.2) \cdot 10^{-4}$	$\mathbf{0.0}$	$1.4 \cdot 10^{-4}$	$1.4 \cdot 10^{-4}$
2005 QK76	$(4.3 \pm 0.9) \cdot 10^{-5}$	$\mathbf{0.0}$	$4.1 \cdot 10^{-5}$	$4.3 \cdot 10^{-5}$
2007 KO4	$(7.3 \pm 4.0) \cdot 10^{-7}$	$\mathbf{0.0}$	$\mathbf{2.2 \cdot 10^{-6}}$	$9.7 \cdot 10^{-7}$
Apophis 2036	$(1.4 \pm 0.8) \cdot 10^{-5}$	$\mathbf{0.0}$	$\mathbf{0.0}$	$1.2 \cdot 10^{-5}$
2010 RF12	$(0.0 \pm 1.7) \cdot 10^{-6}$	$\mathbf{5.1 \cdot 10^{-2}}$	$\mathbf{4.9 \cdot 10^{-2}}$	0.0

Interesting observation

Even if we are mistaken by 50 days in the time of a possible collision the improved approach computes the probabilities accurately!
(need 5 iteration)

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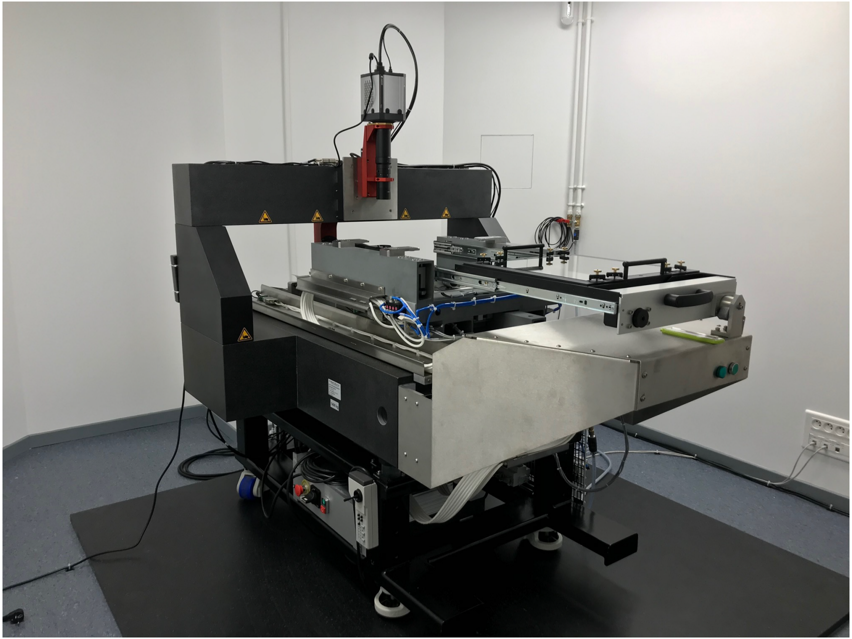
2. Finding known NEOs on old photographic plates (precovery)

NAROO machine



Photographic plates

- New Astrometric Reduction of Old Observations (NAROO) program
- About 22000 photographic plates
- From 1935 until 2002
- From: Observatoire de la Côte d'Azur Nice (OCA Nice), OCA Caussols, Palomar Mountain, European Southern Observatory La Silla, Laboratoire d'Astrophysique de Bordeaux.
- Maximal visual magnitude $\lesssim 20^m$
- Field of View of the plates $\approx 6^\circ$
- The scanned image is about 11 Gb!

Can be known asteroids there!

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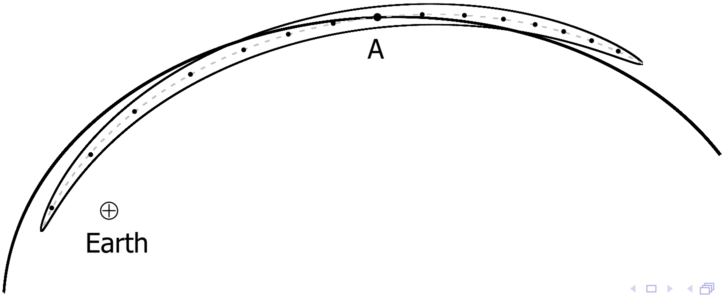
Finding observation on photographic plates

- If the orbit of an asteroid is not well precise it is difficult to associate it with old observations.
- Because of the uncertainty the nominal asteroid's position on the sky can be far from the real one (not on the plate)
- Modify Partial Banana Mapping to this problem.
- Instead of projection onto the target plane here we will project onto the celestial sphere.
- Take 51 virtual asteroids on the main line of curvilinear uncertainty region.
- Map each of them with uncertainty ellipsoids and direction of movement



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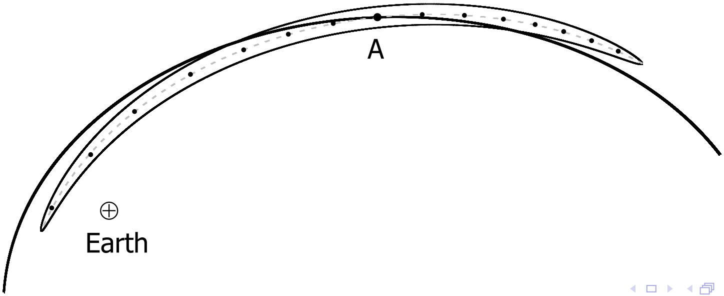

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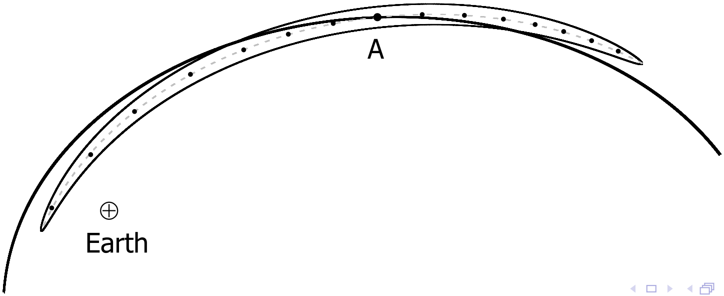

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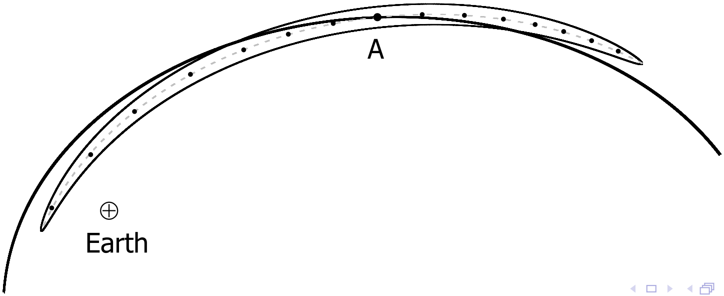

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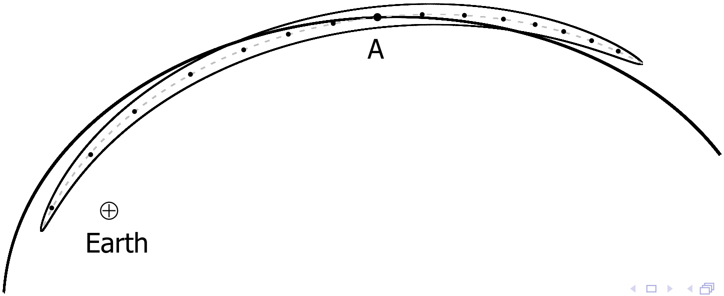

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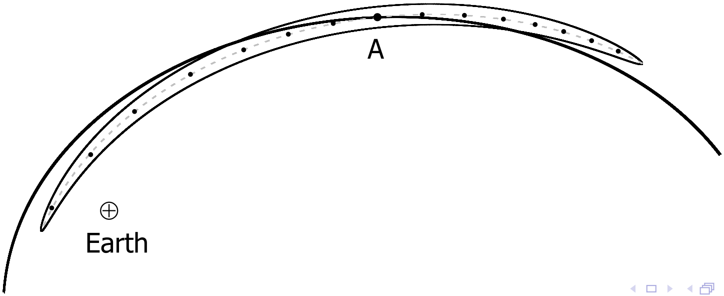
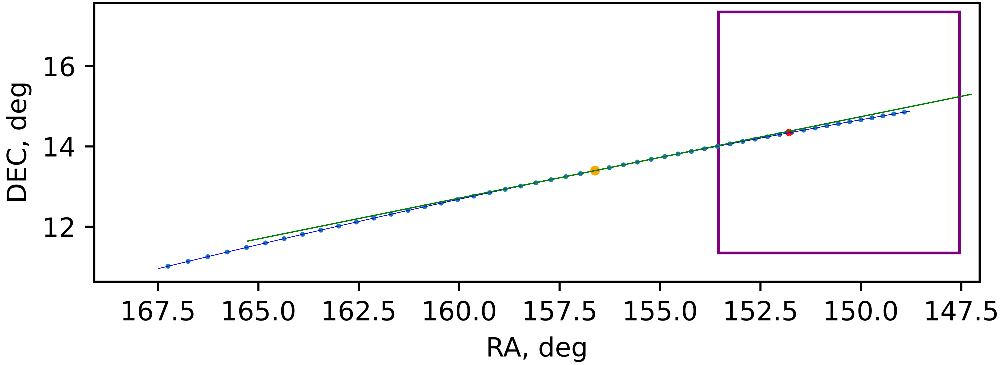


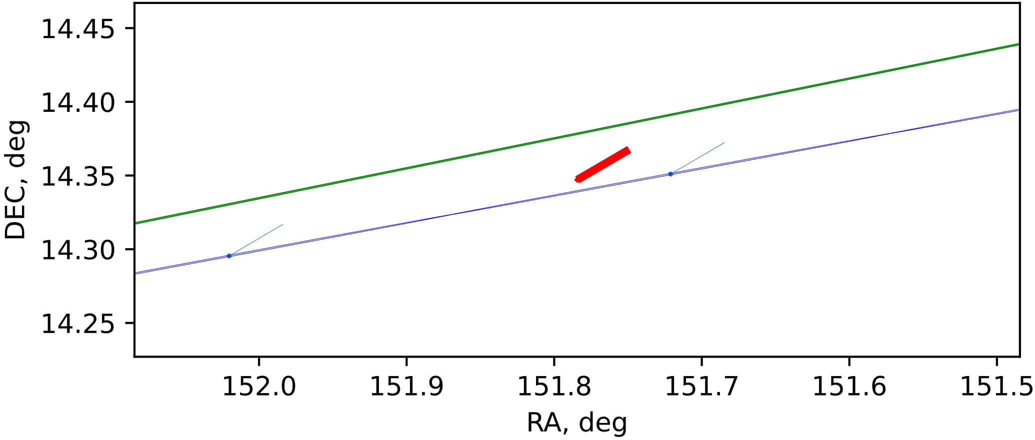
Results for 2015 UM67 precovery on 3 March 1990.

The orange dot is the nominal position of the object.
Blue dots and lines — possible positions of the object at that time.
Red line — actual observation.
Green line — uncertainty ellipse computed from covariance matrix in RA and DEC



Precovery for 2013 NJ10

Blue dots and lines — possible positions of the object at that time.
Red line — actual observation.



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Preliminary results

- Compute the probability that an asteroid can be found on the plate.
- Compute the length of the uncertainty and the visual magnitude.
- Choose the ones with higher probability, higher uncertainty and lower magnitude.

Asteroid	Date	Probability	Magnitude	Length, arcsec
1999GL4	1995-Oct-19 08:33:00	83.7	18.6	77939.3
2014PL51	1954-Sep-06 10:19:12	55.1	17.4	111613.6
2018VQ3	1988-Jun-13 04:48:00	97.8	18.9	79767.9
2016HP19	1950-04-13 08:31:32	100.0	19.0	12137.2
2019BJ5	1954-Jun-28 07:21:58	45.9	16.7	33981.0
*2010EX11	1953-Mar-14 10:35:00	86.4	19.0	2556.0

*We hope what we found is this asteroid.

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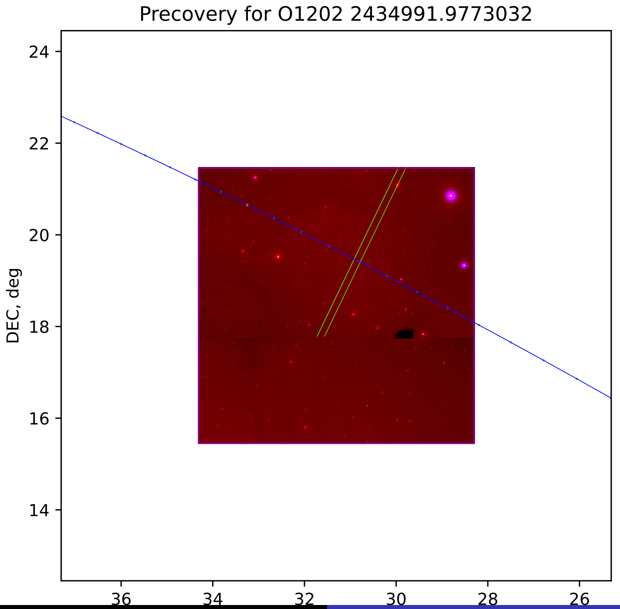
Precovery for 2014PL51



Conclusion

- Partial banana mapping is better than other linear methods for impact probability computation.
- Partial banana mapping method is suitable for precovery of asteroids.

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Thank you for attention!

Introduction to impact monitoring

There are monitoring systems that calculate impact probabilities of asteroids with the Earth: CNEOS, NASA Jet Propulsion Laboratory and NEOCC, European Space Agency.

On April 11, 2024:

- ESA has 1601 Near-Earth Objects
- NASA has 1716 Near-Earth Objects
- Sometimes both systems have different number of possible collisions and different impact probability values for the same asteroid.

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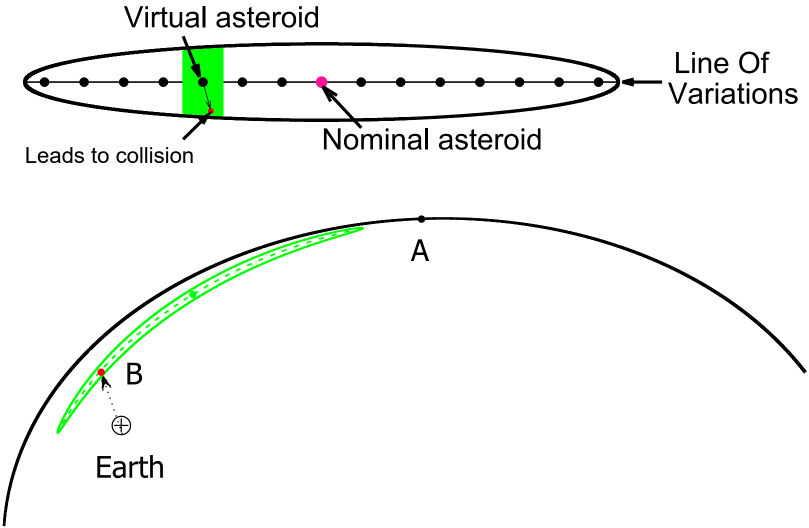
NEOForCE: scheme

Apply Partial Banana Mapping to each virtual asteroid on the Line of Variations.



Results. Asteroid 2008 JL3

Date	NEOForCE	NASA	ESA
2027-05-01	3.3e-4	1.5e-4	1.49E-4
2029-04-30	3.6e-8	-	-
2030-04-30	-	3.0e-8	1.59E-8
2032-04-30	6.6e-8	-	-
2034-04-30	3.6e-7	1.6e-7	-
2036-04-30	1.7e-8	-	-
2038-04-30	1.2e-8	-	-
2039-05-01	1.8e-8	-	-
2042-05-01	5.4e-9	-	-
2043-05-01	1.0e-8	4.6e-9	-
2044-04-30	1.6e-8	-	-
2046-04-30	7.4e-9	-	-
2047-04-27	3.9e-8	3.2e-9	-
2047-04-27	1.2e-7	-	-
2049-04-30	1.2e-8	-	-
2050-04-30	7.9e-9	-	-
2051-05-01	1.7e-7	-	-
2053-04-30	8.7e-9	-	-
2056-04-30	9.6e-9	-	-
2058-05-01	6.8e-9	-	-
2063-05-02	4.7e-7	2.1e-7	-
2065-04-27	4.7e-8	7.9e-8	-

Date	NEOForCE	NASA	ESA
2067-04-30	1.1e-8	-	-
2071-04-30	1.1e-6	2.6e-6	3.04E-6
2075-05-02	1.9e-7	1.0e-7	7.32E-8
2080-04-27	1.6e-8	2.4e-9	-
2081-05-01	3.1e-8	-	-
2090-05-02	1.8e-7	-	-
2090-05-01	8.1e-7	-	7.19E-7
2090-05-01	1.2e-6	-	5.38E-7
2090-05-01	1.3e-6	-	6.31E-7
2092-04-28	1.0e-8	-	-
2093-05-01	3.2e-8	-	-
2094-04-28	2.0e-8	-	-
2094-04-30	7.0e-9	-	-
2095-04-30	3.9e-8	-	-
2097-04-27	-	-	1.17E-9
2099-04-30	3.1e-8	-	-
2100-04-27	1.1e-8	-	-
2100-04-27	4.6e-6	-	6.27E-7
2100-04-27	1.9e-6	-	2.37E-6
2102-05-01	3.1e-8	-	-
2103-04-28	1.2e-8	-	-
2105-05-02	2.8e-8	-	-

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