COMPUTING THE IONOSPHERIC TOTAL ELECTRON CONTENT

FROM DATA TO PRODUCTS

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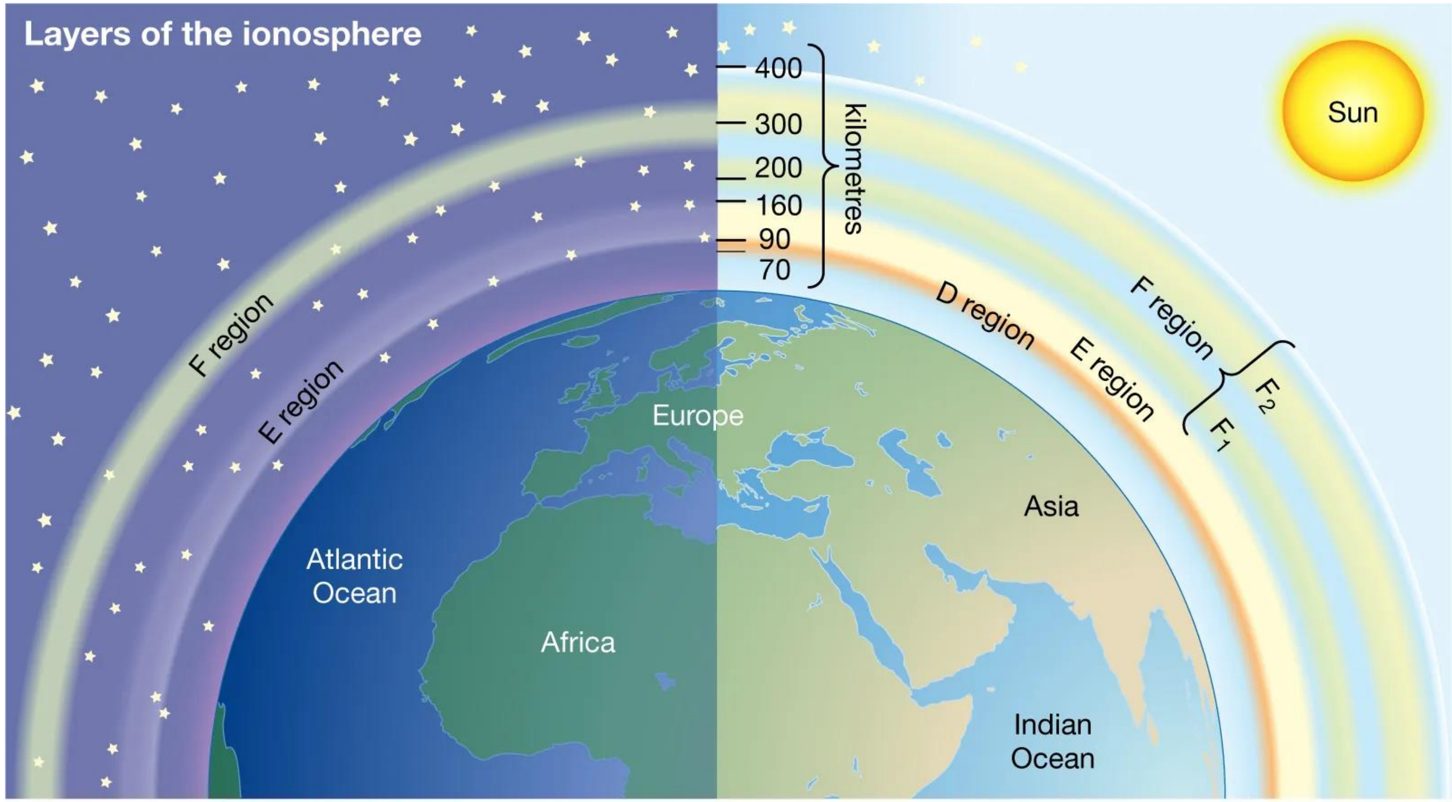
The Ionosphere

The ionosphere is composed of charged particles resulted from the ionization caused by solar radiation.

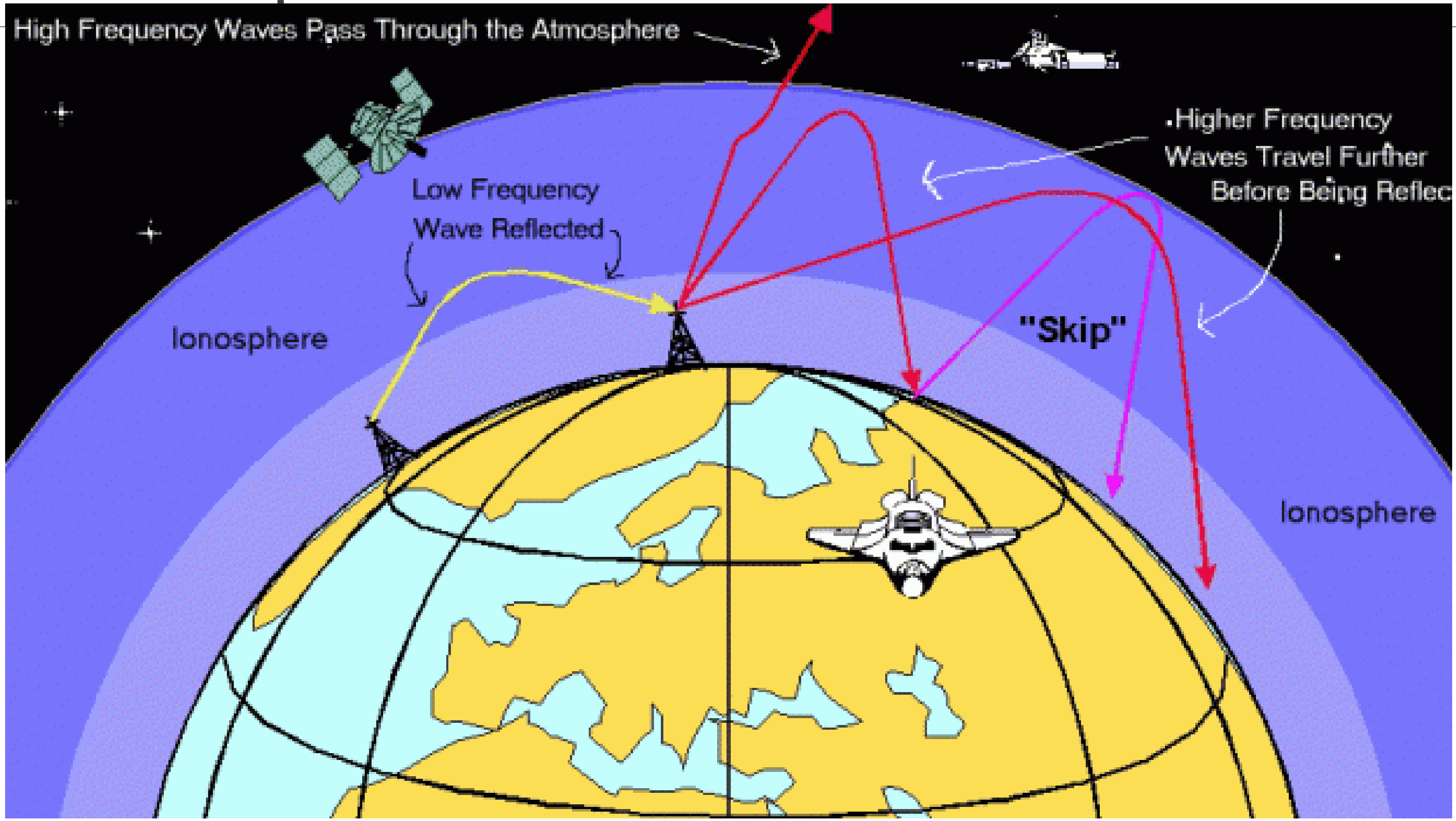
Between 50 and 800 km in altitude.

It reflects and modifies radio waves used for communication and navigation.

The Ionosphere



The Ionosphere



The Satellites

There are 4 GNSS networks active with a total of over 80 satellites.

Each satellites provides observational and navigational data over two channels of different frequencies.

Each satellite ultimately represents a point on the TEC map.

The Total Electron Content

The Total Electron Content is calculated based on the delay difference between two signals.

$$\Delta^{iono} = \pm\frac{40.308}{f^2} * \int_{\overrightarrow{r_T}}^{\overrightarrow{r_R}} N_e dl$$

- 40.308 represents the refractive index of the ionosphere
- f represents a combination term of the two frequency values
- the integral represents the total number of electrons along the ray path

The Observable Data

TEC can be calculated using either the pseudorange or the phase.

The pseudo-range (PR) is the distance from the receiver antenna to the satellite antenna including receiver and satellite clock offsets (and other biases, such as atmospheric delays).

The phase is the carrier-phase measured in whole cycles.

The Observable Data

```
     3.02           OBSERVATION DATA    M: MIXED            RINEX VERSION / TYPE
Spider V6.2.0.6944  NIEP                20190321 130926 UTC PGM / RUN BY / DATE
SNR is mapped to RINEX snr flag value [1-9]                 COMMENT
LX:     < 12dBHz -> 1; 12-17dBHz -> 2; 18-23dBHz -> 3       COMMENT
       24-29dBHz -> 4; 30-35dBHz -> 5; 36-41dBHz -> 6       COMMENT
       42-47dBHz -> 7; 48-53dBHz -> 8; >= 54dBHz -> 9       COMMENT
BAES                                                        MARKER NAME
BAES                                                        MARKER NUMBER
Eduard Nastase      National Institute for Earth Physics    OBSERVER / AGENCY
496841              LEICA GRX1200+GNSS  8.71/6.110          REC # / TYPE / VERS
19132013            LEIAR10         NONE                    ANT # / TYPE
  4217705.7411  1819145.1852  4410495.1603                  APPROX POSITION XYZ
        0.0000        0.0000        0.0000                  ANTENNA: DELTA H/E/N
G   16 C1C L1C D1C S1C C2S L2S D2S S2S C2W L2W D2W S2W C5Q  SYS / # / OBS TYPES
       L5Q D5Q S5Q                                          SYS / # / OBS TYPES
R   12 C1C L1C D1C S1C C2P L2P D2P S2P C2C L2C D2C S2C      SYS / # / OBS TYPES
DBHZ                                                        SIGNAL STRENGTH UNIT
     1.000                                                  INTERVAL
  2019    03    21    12    00    0.0000000     GPS         TIME OF FIRST OBS
  2019    03    21    12    59   59.0000000     GPS         TIME OF LAST OBS
     0                                                      RCV CLOCK OFFS APPL
G L2S -0.25000                                              SYS / PHASE SHIFT
G L2X -0.25000                                              SYS / PHASE SHIFT
R L2P  0.25000                                              SYS / PHASE SHIFT
E L8Q -0.25000                                              SYS / PHASE SHIFT
  9 R04  6 R05  1 R06 -4 R13 -2 R14 -7 R15  0 R17  4 R23  3 GLONASS SLOT / FRQ #
    R24  2                                                  GLONASS SLOT / FRQ #
 C1C  -71.940 C1P  -71.940 C2C  -71.940 C2P  -71.940        GLONASS COD/PHS/BIS
    18    18  1929     7                                    LEAP SECONDS
                                                            END OF HEADER
> 2019 03 21 12 00  0.0000000  0 17
G06  24466000.080   128569714.61607      2897.855          43.250   24465998.560   100184186.29907      2258.071          45.500
G10  25100611.780   131904603.05106     -1150.835          40.800
G12  21671910.140   113886786.39308      2378.945          50.000   21671903.060    88742782.92707      1853.726          45.900
G13  23405892.560   122998782.93307     -3729.058          44.000
G15  22275944.260   117060892.70308     -2783.388          49.200   22275938.100    91216264.68707     -2168.874          45.850
G17  22292066.000   117145637.37008     -1968.778          50.100   22292058.100    91282310.48907     -1534.112          46.100
G19  20812962.720   109372884.17908      -168.818          49.000
G24  20419128.140   107303275.70708      1033.576          51.300   20419123.540    83612935.39708       805.385          49.900
G28  24977645.300   131258394.18707     -2825.923          42.600
R04  21881151.940   117172596.27407     -3191.274          44.950   21881149.660    91134220.72807     -2482.104          43.750
R05  21772645.940   116387287.56207        80.237          46.250   21772645.780    90523452.15007        62.406          45.650
R13  21329655.460   113899179.33807     -2326.146          42.750   21329654.520    88588274.37706     -1809.226          39.750
R14  19668929.840   104846487.65308       434.337          49.800   19668929.760    81547284.57607       337.818          46.900
R15  22101985.480   118106323.92807      2804.055          46.200   22101985.300    91860483.67407      2180.934          43.700
R17  23634979.380   126475552.15606      4581.570          38.150   23634995.580    98369934.10105      3563.449          35.150
R23  20634144.180   110378831.13807      -641.818          47.850   20634144.880    85850185.75806      -499.192          41.250
R24  20850498.520   111497017.77007      3027.328          43.450   20850495.440    86719886.66905      2354.591          34.150
```

The Vertical TEC

The TEC value obtained so far is related to the signal path (slant TEC) and still has many biases. In order to remove them we must calculate the vertical TEC.

$$I(t) = S\big(\varepsilon(t)\big) * V(t) + B$$

$$S(\varepsilon) = \frac{1}{\cos[\sin^{-1}\left[\frac{R}{R+h}\cos(\varepsilon)\right]]}$$

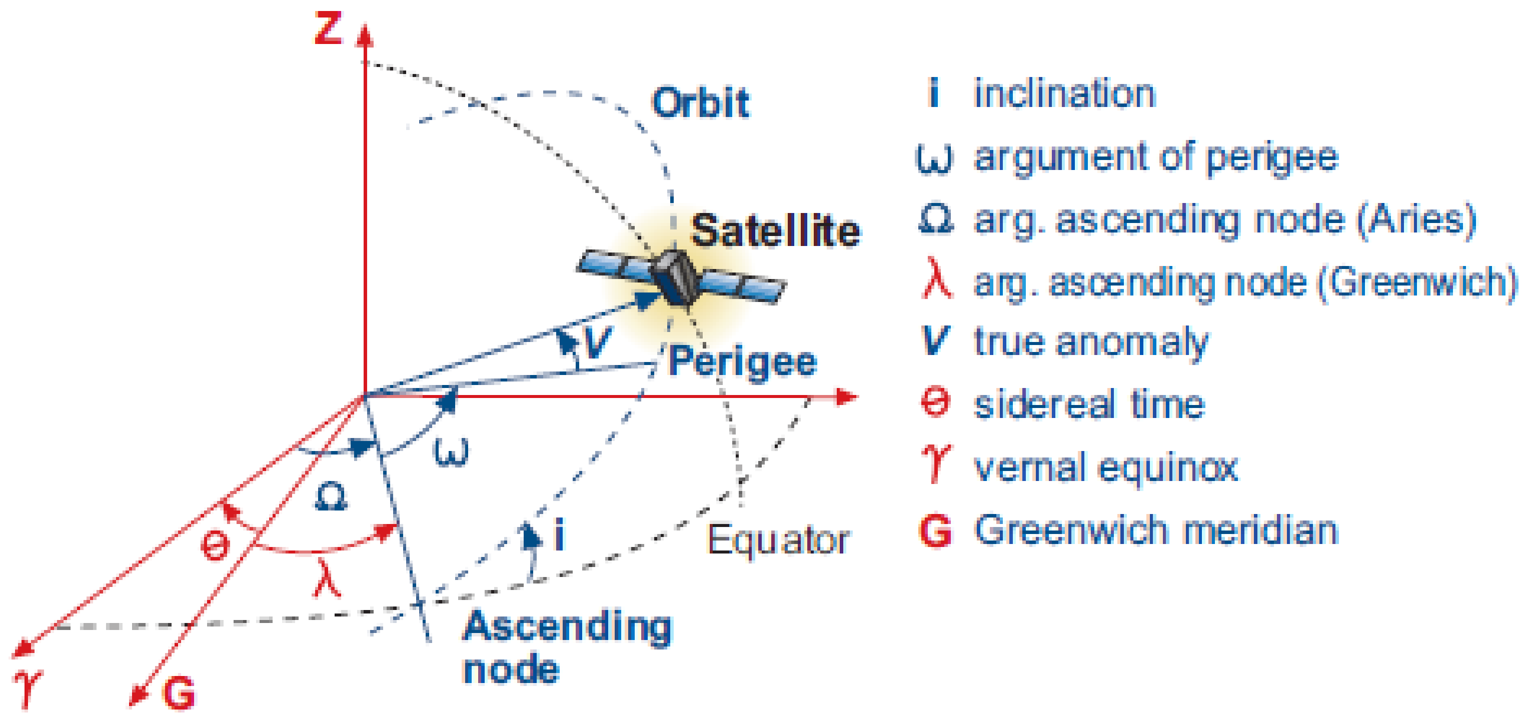
The Position of the Satellites

The exact position of the satellites must be calculated in order to have the satellite's elevation relative to the receptor and the piercing point of the signal.

The position is calculated using the following equation describing an Keplerian orbit, all the arguments for it being transmitted in the navigational data of the satellites.

$$\boldsymbol{r}(t) = \boldsymbol{r}(t, a, ec, i, \Omega, \omega, \tau)$$

The Position of the Satellites



The Navigational Data

```
     3.02           N: GNSS NAV DATA    G: GPS              RINEX VERSION / TYPE
Spider V7.4.0.8125  NIEP                20190929 105505 UTC PGM / RUN BY / DATE
GPSA   8.3819D-09  1.4901D-08 -5.9605D-08 -5.9605D-08       IONOSPHERIC CORR
GPSB   8.3968D+04  1.6384D+04 -1.3107D+05 -6.5536D+04       IONOSPHERIC CORR
GPUT  3.7252902985D-09 7.105427358D-15  61440 2073          TIME SYSTEM CORR
    18    18  1929     7                                    LEAP SECONDS
                                                            END OF HEADER
G01 2019 09 26 15 59 44-1.418734900653D-04-1.296029950026D-11 0.000000000000D+00
     1.800000000000D+01 1.596875000000D+01 4.654479468513D-09-2.863159206485D+00
     7.040798664093D-07 9.116305620410D-03 1.508742570877D-06 5.153643062592D+03
     4.031840000000D+05 4.842877388000D-08 1.173075050256D+00-2.402812242508D-07
     9.768053408461D-01 3.591250000000D+02 7.648625576528D-01-8.342490254165D-09
     1.692927603658D-10 1.000000000000D+00 2.072000000000D+03 0.000000000000D+00
     2.000000000000D+00 0.000000000000D+00 5.587935447693D-09 1.800000000000D+01
     3.959100000000D+05
G02 2019 09 27 04 00 00-3.136969171464D-04-8.071765478235D-12 0.000000000000D+00
     1.010000000000D+02 5.953125000000D+01 4.968064182265D-09-2.568085076469D+00
     3.149732947350D-06 1.917091070209D-02 1.288950443268D-06 5.153537820816D+03
     4.464000000000D+05 1.918524503708D-07 1.101426612645D+00-4.842877388000D-08
     9.559425344551D-01 3.528750000000D+02-1.712013282145D+00-8.339990031914D-09
     3.239420676149D-10 1.000000000000D+00 2.072000000000D+03 0.000000000000D+00
     2.000000000000D+00 0.000000000000D+00-2.048909664154D-08 1.010000000000D+02
     4.391400000000D+05
G03 2019 09 26 18 00 00-1.527043059468D-05-4.320099833421D-12 0.000000000000D+00
     8.600000000000D+01-1.223125000000D+02 4.483401205846D-09-2.933479087727D+00
    -6.407499313354D-06 2.555537386797D-03 8.577480912209D-06 5.153710905075D+03
     4.104000000000D+05-3.539025783539D-08 2.215335660114D+00-1.154839992523D-07
     9.637366562315D-01 2.143750000000D+02 7.513666497569D-01-8.124981576429D-09
    -3.675152959737D-10 1.000000000000D+00 2.072000000000D+03 0.000000000000D+00
     2.000000000000D+00 0.000000000000D+00 1.862645149231D-09 8.600000000000D+01
     4.031400000000D+05
```

The Position Calculation Algorithm

Compute the time t_k from the ephemerides reference epoch t_{oe} (t and t_{oe} are expressed in seconds in the GPS week):

$$t_k = t - t_{oe}$$

Compute the mean anomaly for t_k:

$$M_k = M_0 + (\frac{\sqrt{\mu}}{\sqrt{a^3}} + \Delta n)$$

The Position Calculation Algorithm

Solve (iteratively) the Kepler equation for the eccentric anomaly E_k:

$$M_k = E_k - ec * \sin E_k$$

Compute the true anomaly vk:

$$v_k = \tan^{-1}\left(\frac{\sqrt{1 - ec^2} * \sin E_k}{\cos E_k - ec}\right)$$

The Position Calculation Algorithm

Compute the argument of latitude u_k from the argument of perigee ω, true anomaly v_k and corrections c_{uc} and c_{us}:

$$u_k = \omega + v_k + c_{uc} \cos 2(\omega + v_k) + c_{us} \sin 2(\omega + v_k)$$

Compute the radial distance r_k, considering corrections c_{rc} and c_{rs}:

$$r_k = a(1 - ec * \cos E_k) + c_{rc} \cos 2(\omega + v_k) + c_{rs} \sin 2(\omega + v_k)$$

The Position Calculation Algorithm

Compute the inclination i_k of the orbital plane from the inclination i_o at reference time t_{oe}, and corrections c_{ic} and c_{is}:

$$i_k = i_0 + it_k + c_{ic} \cos 2(\omega + v_k) + c_{is} \sin 2(\omega + v_k)$$

Compute the longitude of the ascending node λ_k:

$$\lambda_k = \Omega_0 + (\dot{\Omega} - \omega_E)t_k - \omega_E t_{oe}$$

The Position Calculation Algorithm

Compute the coordinates in the TRS frame, applying three rotations (around u_k, i_k and λ_k):

$$\begin{pmatrix} X_k \\ Y_k \\ Z_k \end{pmatrix} = R_3(-\lambda_k) * R_1(-i_k) * R_3(-u_k) * \begin{pmatrix} r_k \\ 0 \\ 0 \end{pmatrix}$$

$$R_1(x) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos x & \sin x \\ 0 & -\sin x & \cos x \end{pmatrix}$$

$$R_3(x) = \begin{pmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Removing the biases

$$I(t) = S\big(\varepsilon(t)\big) * V(t) + B$$

Out of all the biases, the most significant one, because of how hard it is to characterize, is the Differential Code Bias (DCB).

The DCB is caused by the electronics of the satellite and the receiver and is considered constant over the course of up to a few days.

Removing the biases

For a numerical solution, we need another starting presumption about the biases or the slant TEC in order to be able to calculate them:

- The sum of all biases in a GNSS network is zero;
- The average TEC value of a set area covered by a single receiver recorded by any satellite is constant over a set time interval (Otsuka method);

The Otsuka Method

It was used for a higher spatial resolution TEC map over Japan.

It is based on partially deriving the following estimator:

$$E = \sum_{i}^{N_s} \sum_{k}^{N_t} W_k^i [\overline{I_k / S\left(\varepsilon_k^i\right)} - \left(\overline{V_k} + \overline{1/S\left(\varepsilon_k^i\right)}\, B^i\right)]^2$$

$$W_k^i = \overline{1/S\left(\varepsilon_k^i\right)}$$

The Otsuka Method

$$W_1^1 T_1^1 + W_1^2 T_1^2 - V_1 \left(W_1^1 + W_1^2\right) - \left({W_1^1}^2 B^1 + {W_1^2}^2 B^2\right) = 0$$

$$W_2^1 T_2^1 + W_2^2 T_2^2 - V_2 \left(W_2^1 + W_2^2\right) - \left({W_2^1}^2 B^1 + {W_2^2}^2 B^2\right) = 0$$

$${W_1^1}^2 T_1^1 + {W_2^1}^2 T_2^1 - V_1 {W_1^1}^2 - V_2 {W_2^1}^2 - B^1 \left({W_1^1}^3 + {W_2^1}^3\right) = 0$$

$${W_1^2}^2 T_1^2 + {W_2^2}^2 T_2^2 - V_1 {W_1^2}^2 - V_2 {W_2^2}^2 - B^2 \left({W_1^2}^3 + {W_2^2}^3\right) = 0$$

Generating TEC Maps

For regional maps, a sufficiently spread set of receivers should provide enough points to plot a map.

For local maps, the higher the spatial resolution, the more permissive the interpolation/ fit method must be.

ESA provides TEC maps with a spatial resolution of 1° x 1° longitude and latitude.

Radial Basis Function Interpolation

One very permissive interpolation method is radial basis function interpolation.

X		Y
	Z	

Radial Basis Function Interpolation

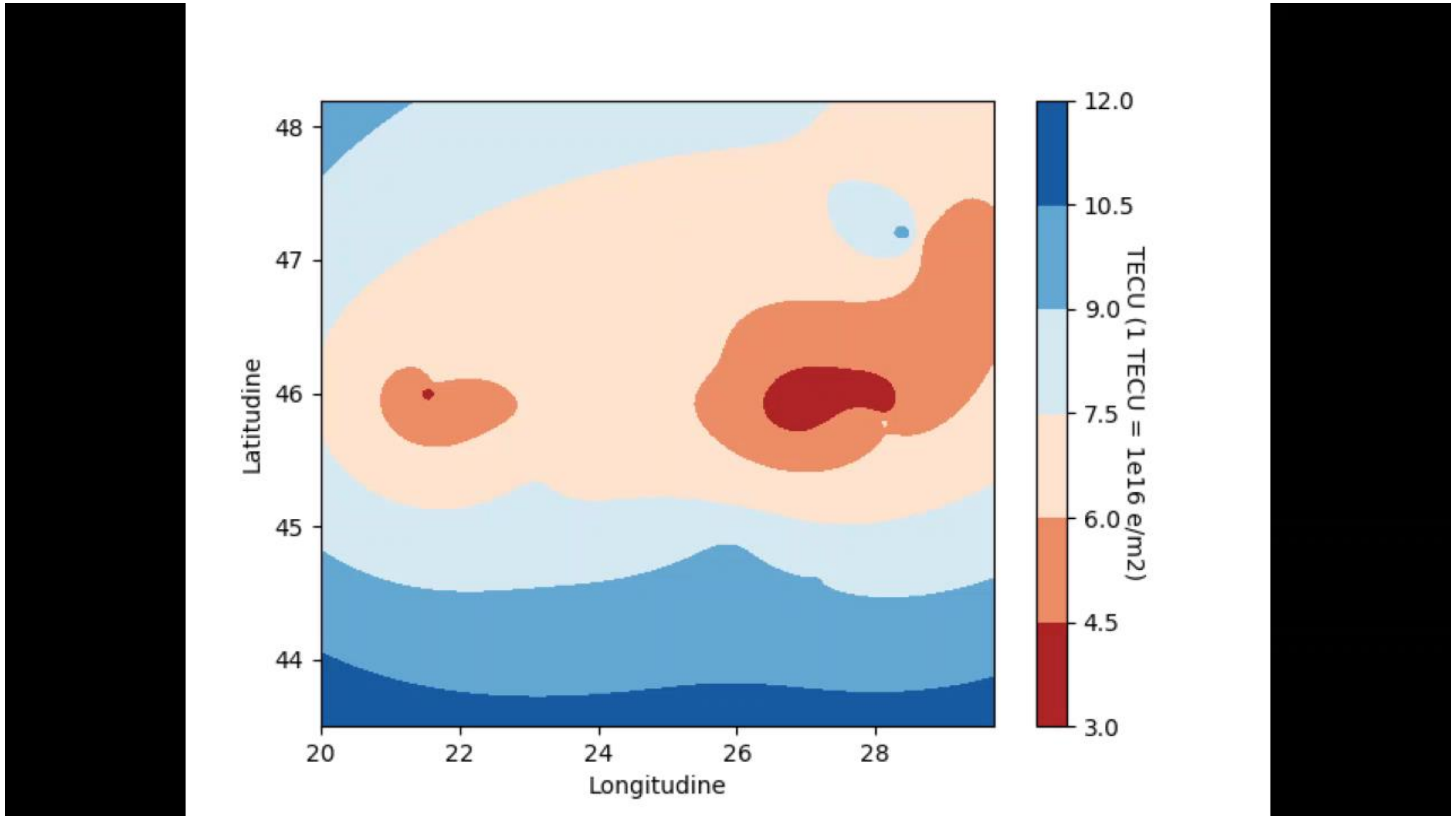
$$y(x) = \sum_{i=0}^{N-1} w_i \phi(|x - x_i|)$$

$$\phi(r) = (r^2 + {r_0}^2)^{\alpha}$$

The values chosen for r_0 and α have been 0, 0.375 respectively.

TEC Maps

The TEC maps below cover the entire surface of Romania and it was calculated for 27.09.2020, in the time interval 19:00 - 21:00:



Thank you for your attention!