From Saturn's strong tides to the ocean of Mimas and beyond

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- LTE/Paris Observatory -

26/03/2025 Astronomical Institute

Outline:

I - The Saturnian system in three « simple » questions

II - Brief recall on tides

III - Answering question 1

IV - Answering question 2

V - Answering question 3

Part I
The Saturnian system in three « simple » questions

The relative size of the main moons – Question 1

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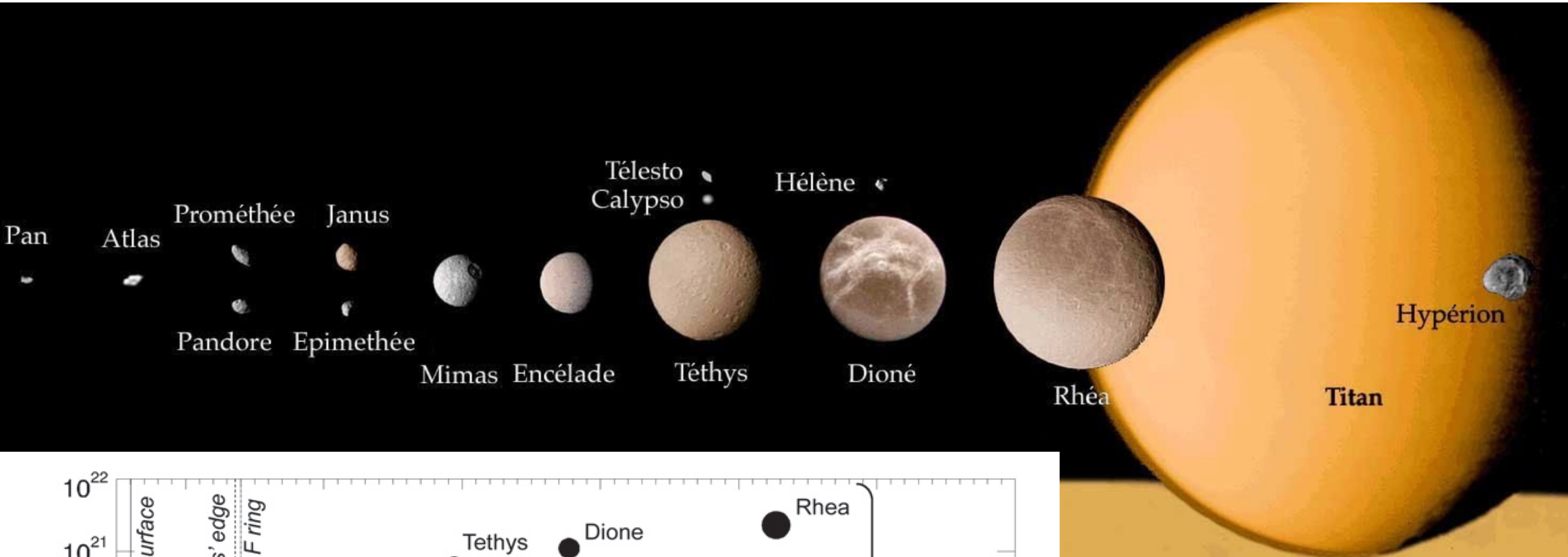
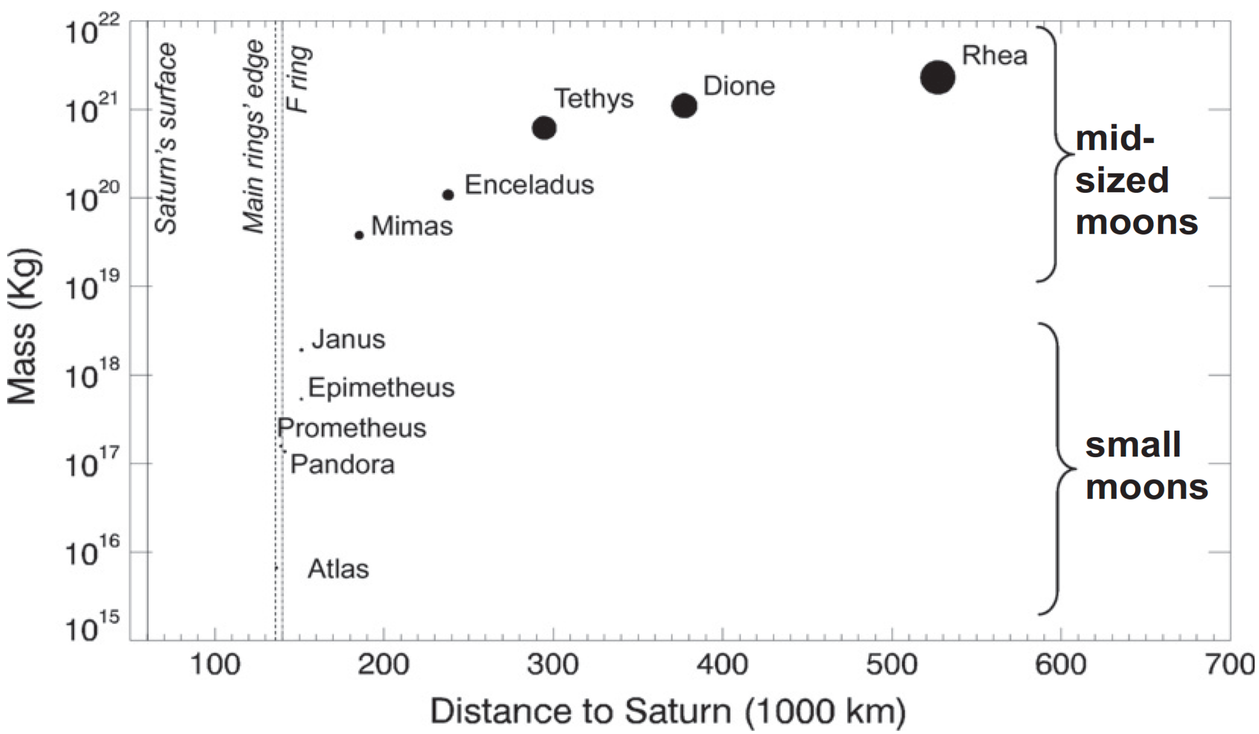
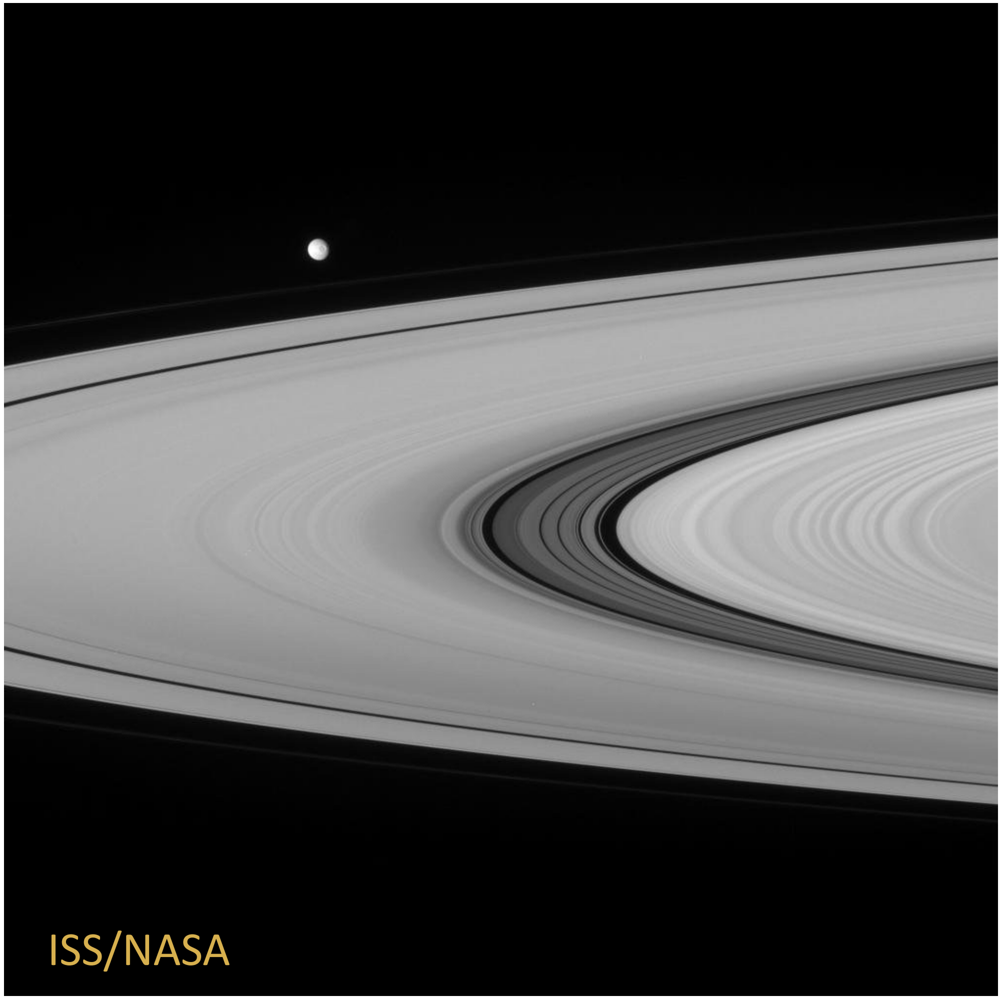




Charnoz et al. 2011

The Cassini division – Question 2



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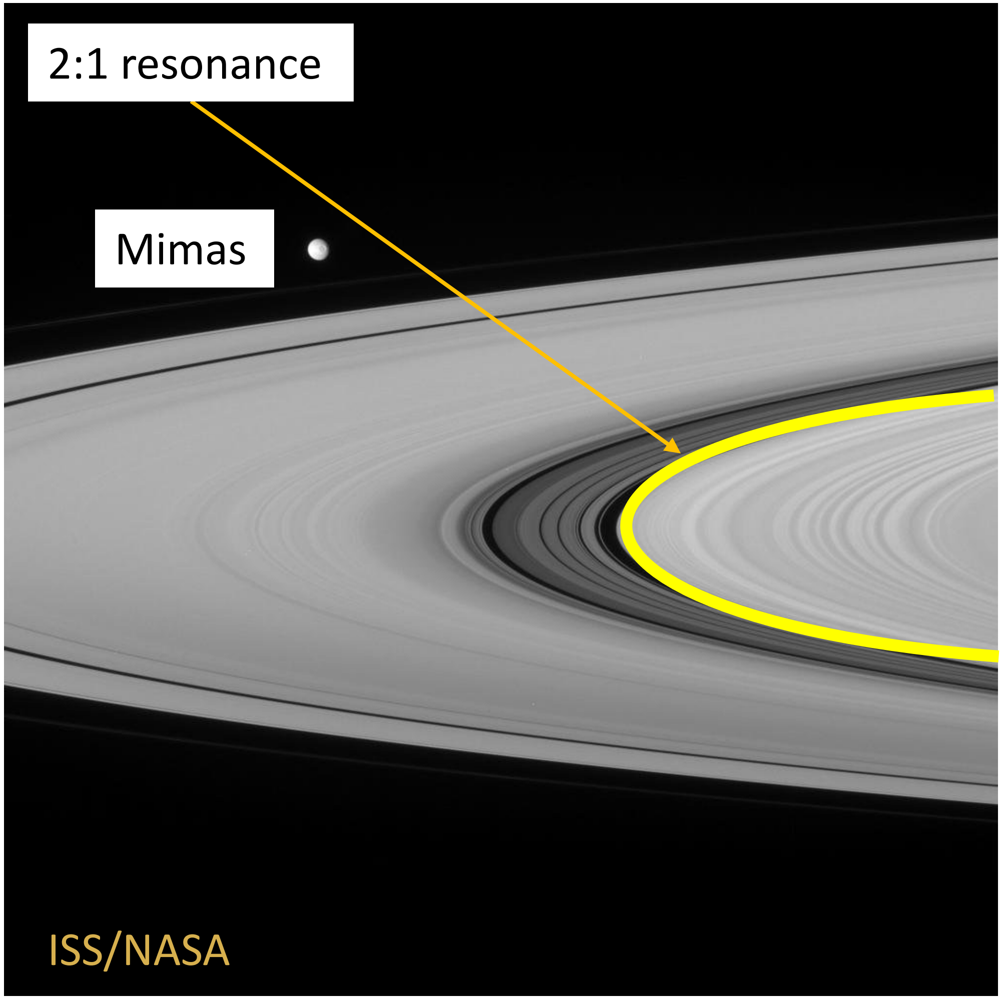

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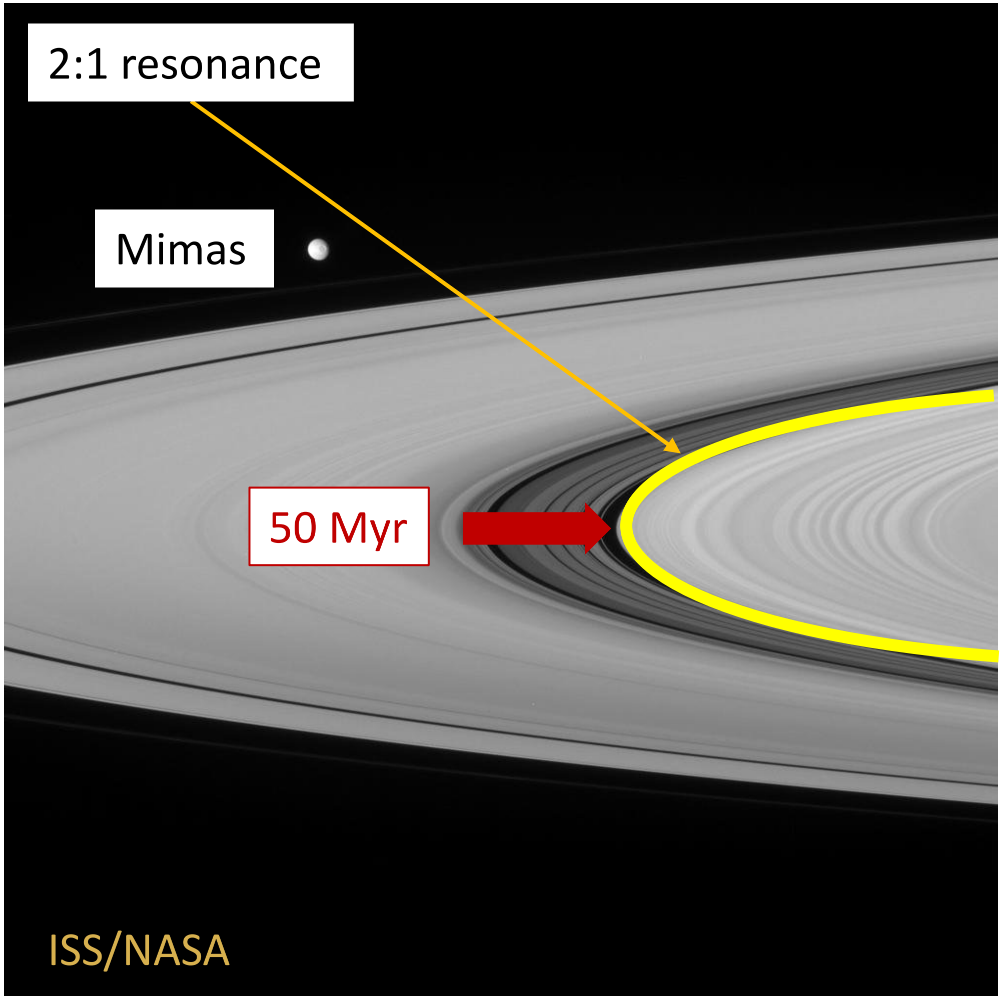
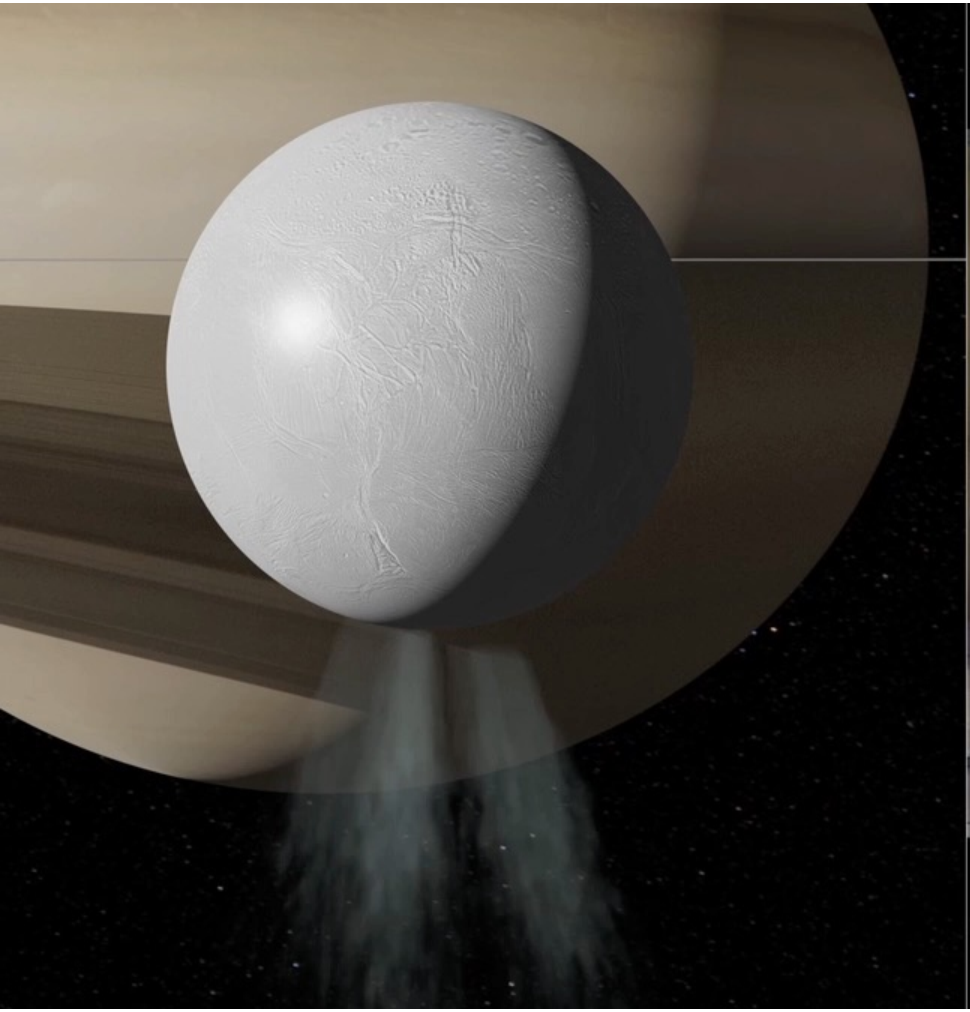
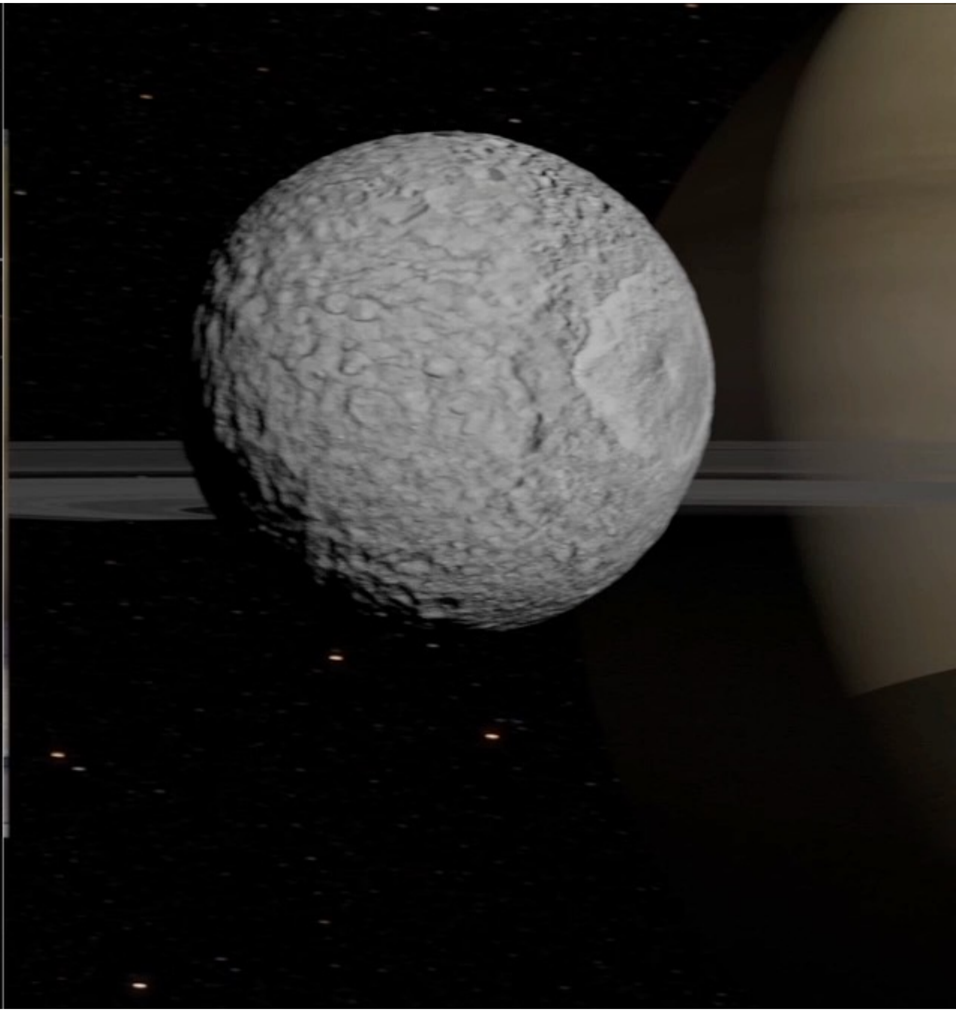


The Mimas / Enceladus paradox – Question 3



Enceladus

R = 252.1 km
a = 237,948 km
e = 0.0047



Mimas

R = 198.2 km
a = 185,539 km
e = 0.0196

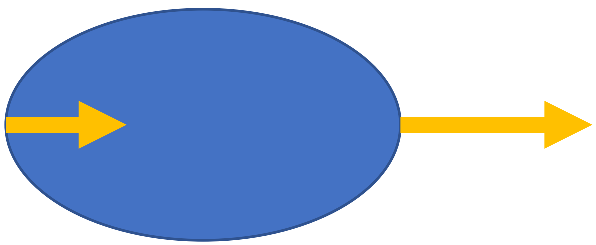
Part II

Brief recall on tides

Brief recall on tides

Gravitation law:

$$F = -\frac{GMm\vec{\boldsymbol{r}}}{r^3}$$



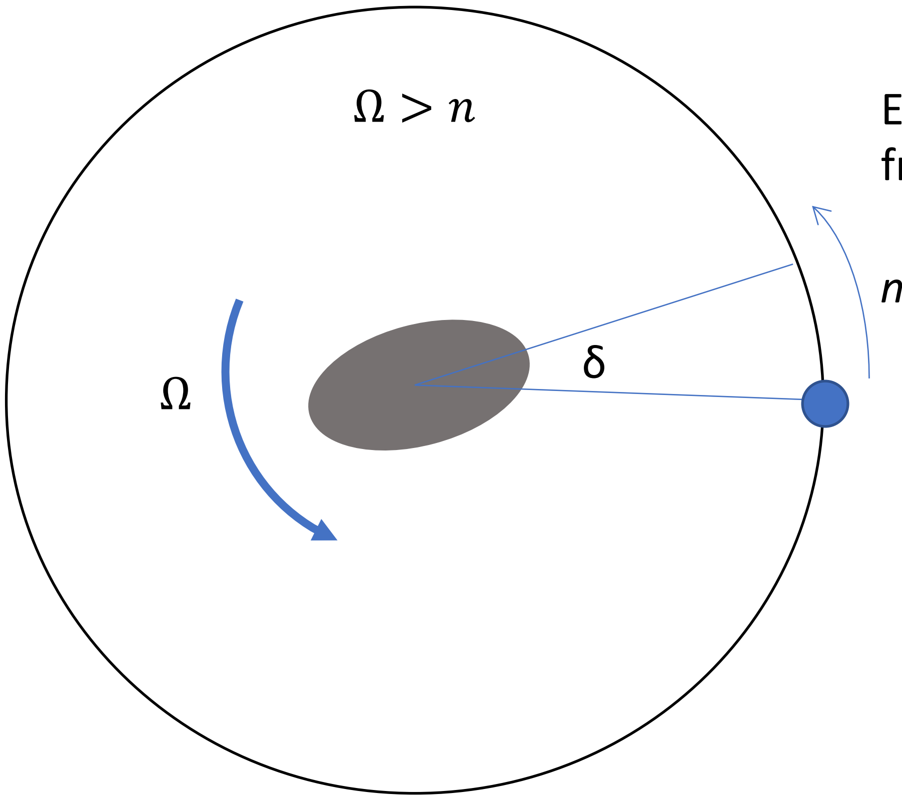
Tidal effects are a *differential gravitational effect*, induced by a massive object, on an extended body.

Such a deformation entails frition, and as a consequence heat production

<u>Brief recall on tides</u>

Let's consider a moon raising tides on Saturn

Dissipation implies time lag Δt which is connected to the geometrical lag δ by: $2(\Omega - n)\Delta t = \delta$



Exchange of angular momentum and energy from Saturn's spin into the orbit

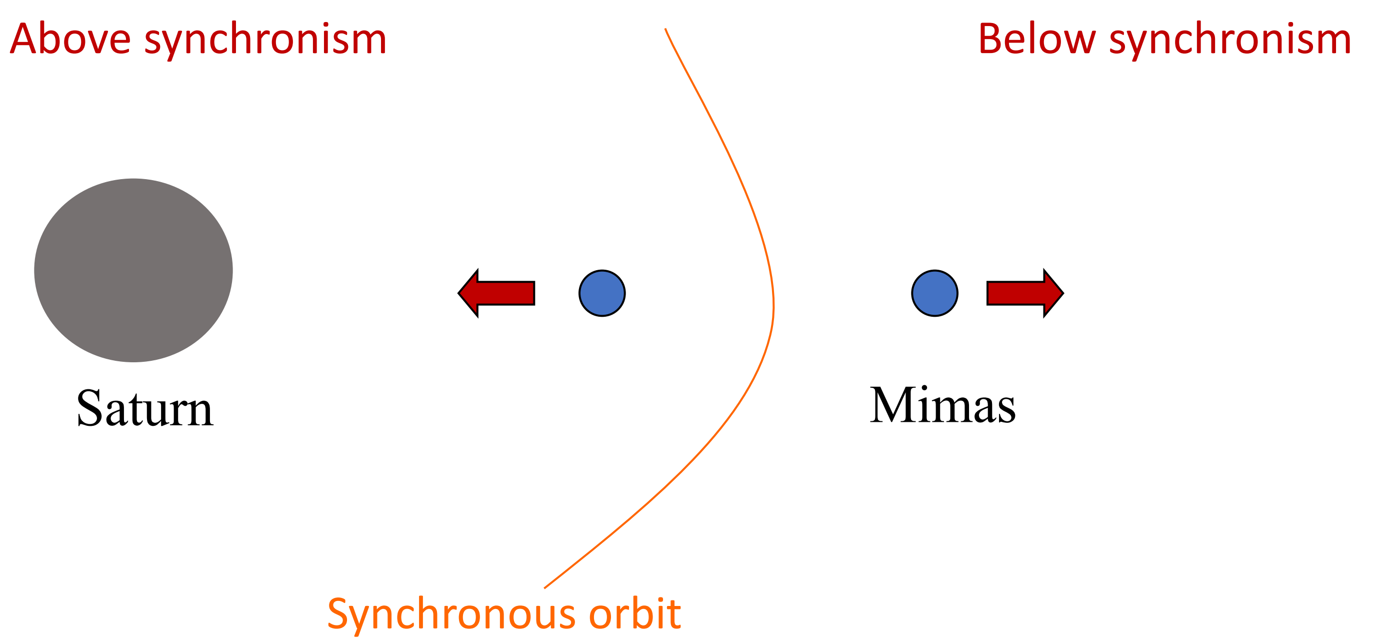
$$\frac{da}{dt} = \frac{3k_2 m}{QM}\left(\frac{Er}{a}\right)^5 na$$

Q: tidal quality factor
k_2: Love number

Brief recall on tides

A paper of reference that estimates Q for the giant planets is Goldreich and Soter (1966)

Assuming that the main satellites were formed beyond the synchronous orbit, one can give a higher bound for k_2/Q



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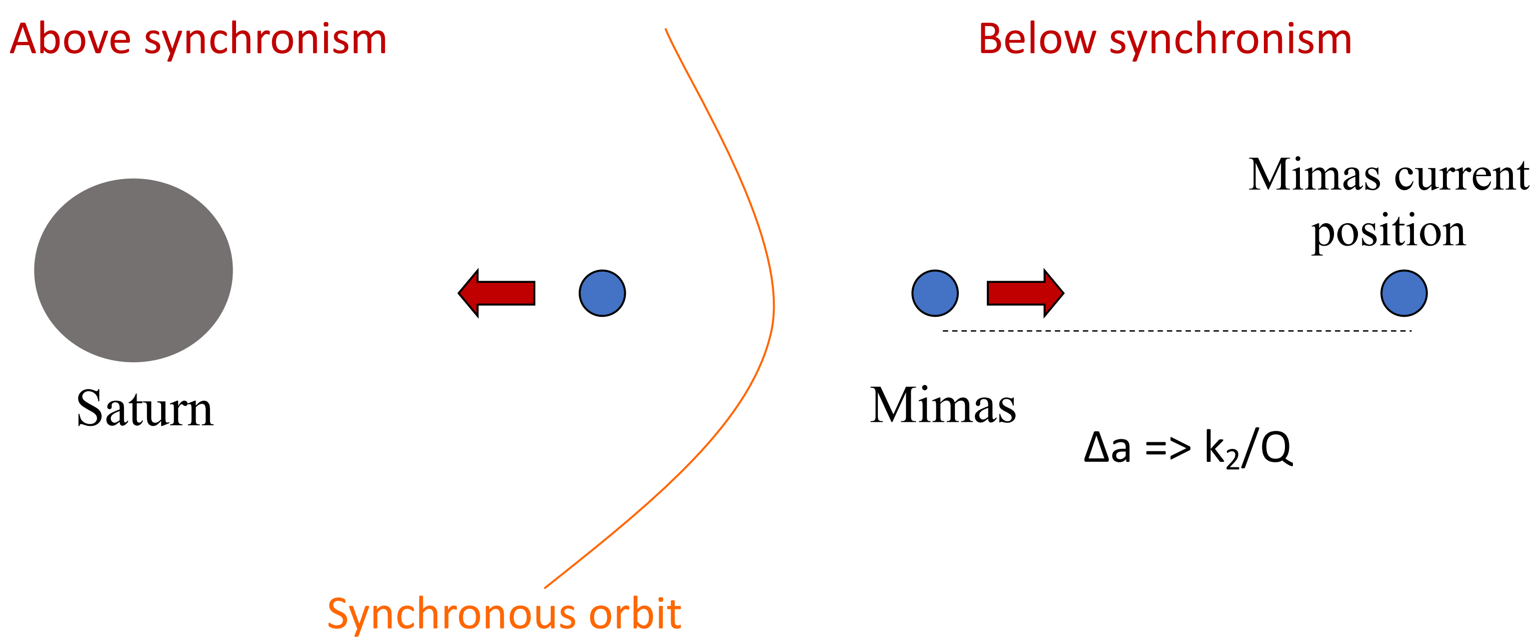
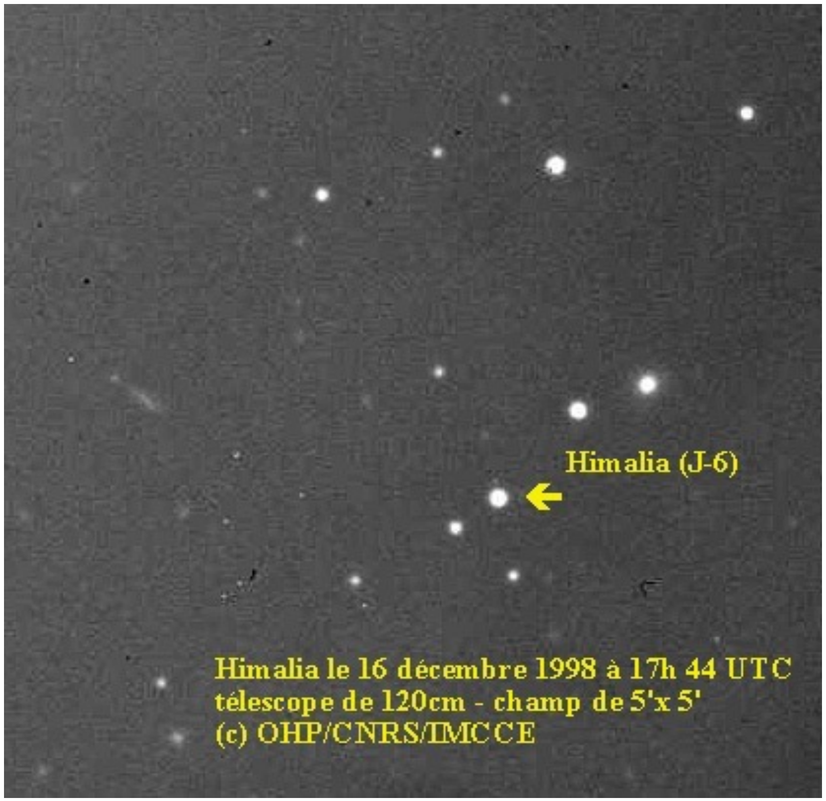
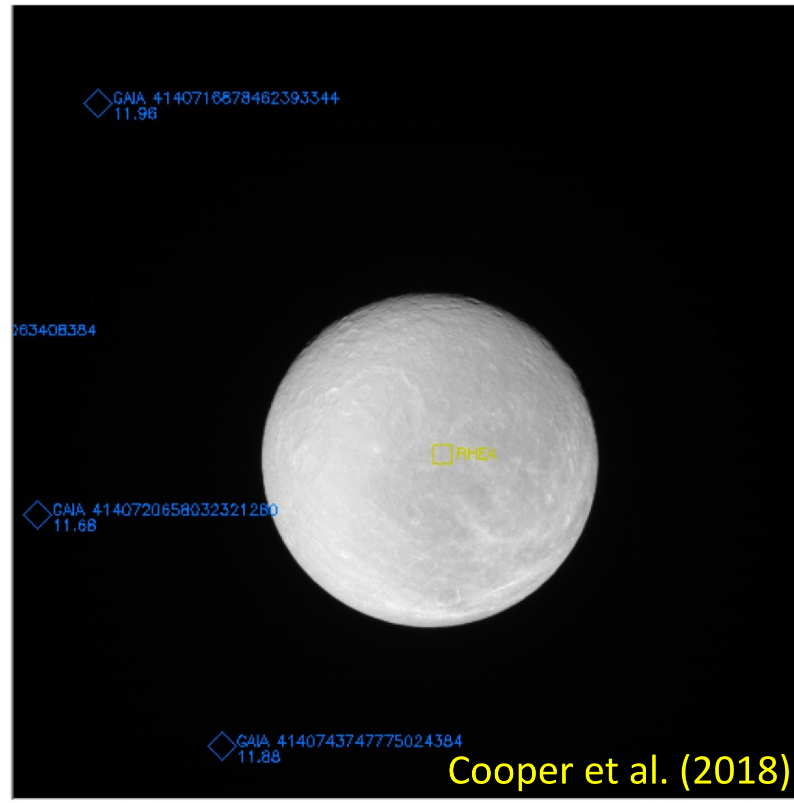


Quantification of tides from astrometry

Astrometry is the discipline that aims to provide positions of celestial objects in space with the highest accuracy.



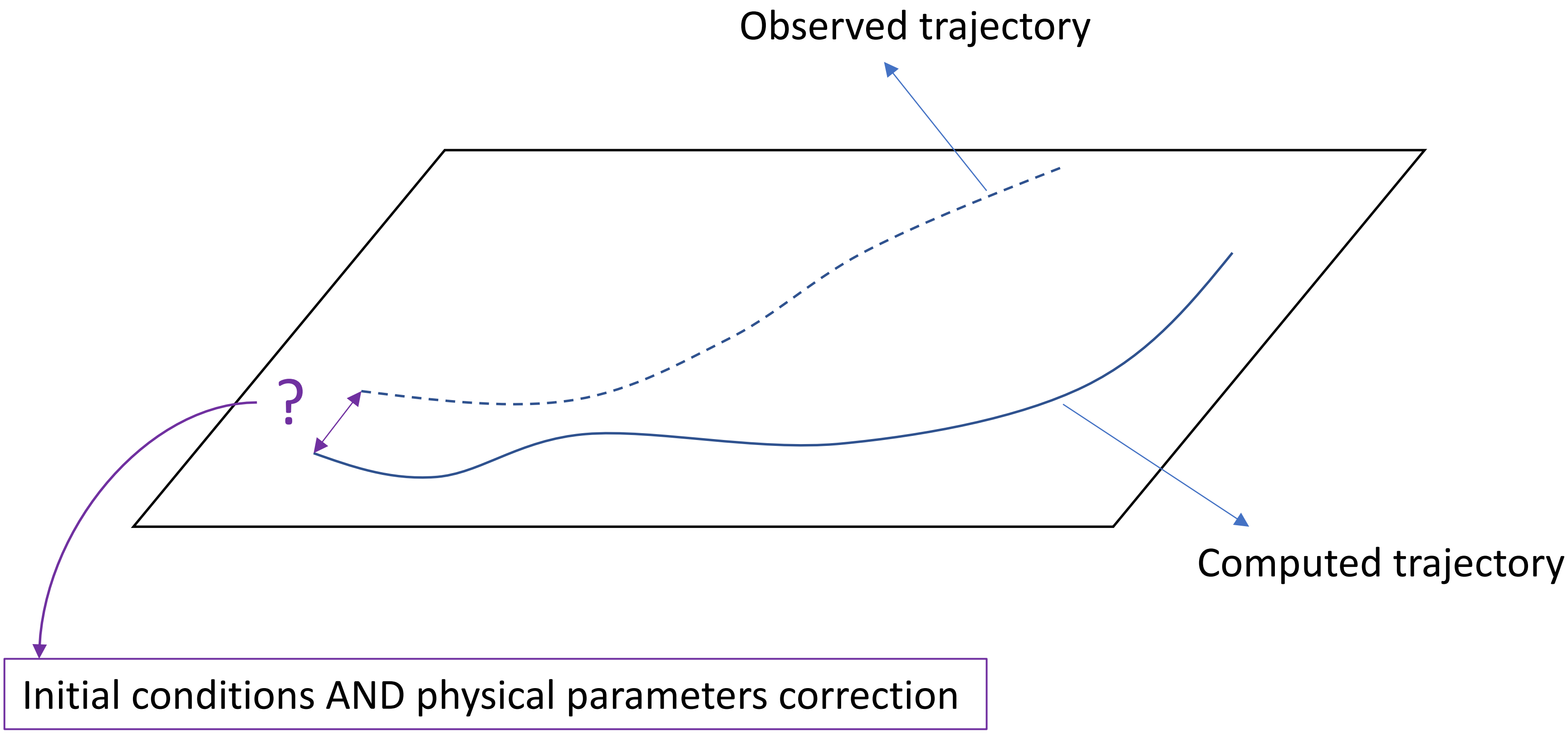
Example 1



Example 2

Uses stars in the background to determine the relation between pixels and angle on the celestial sphere.

Quantification of tides from astrometry



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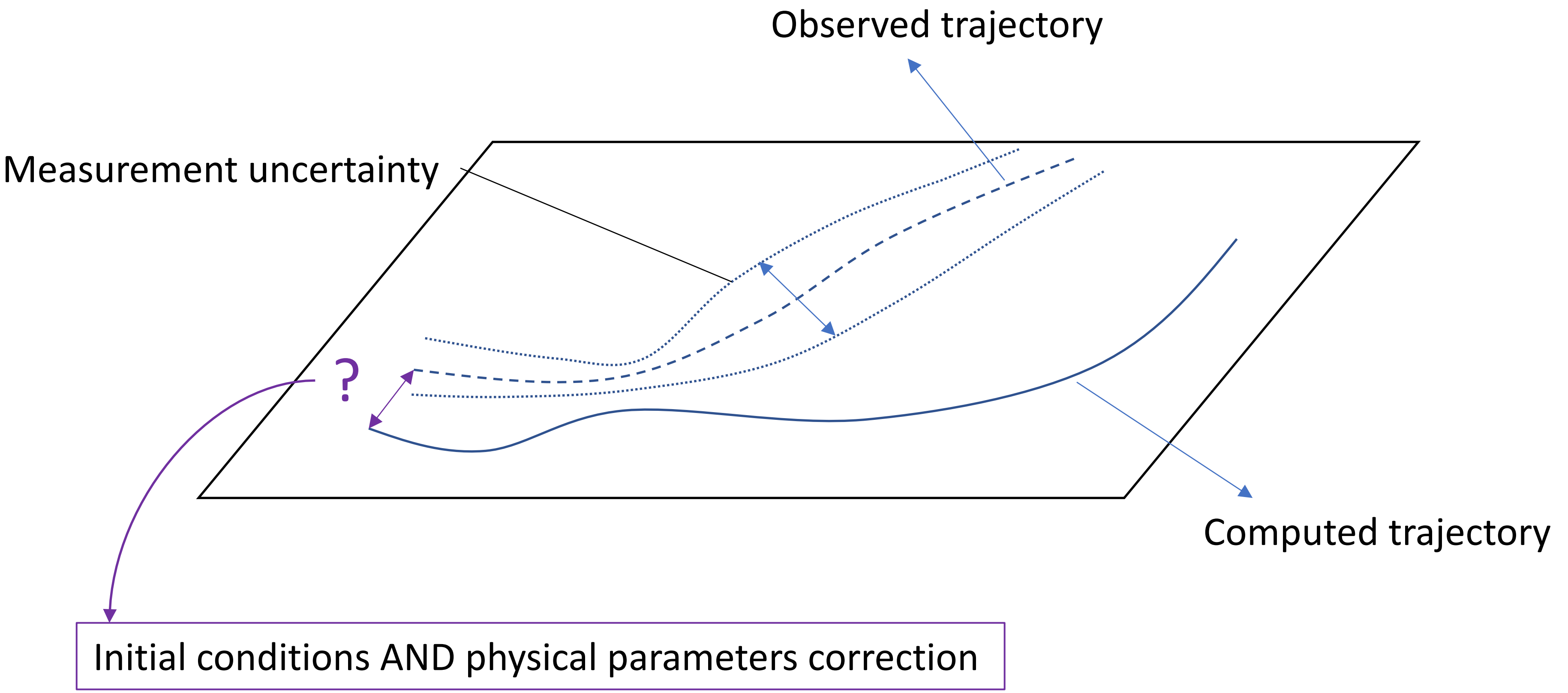

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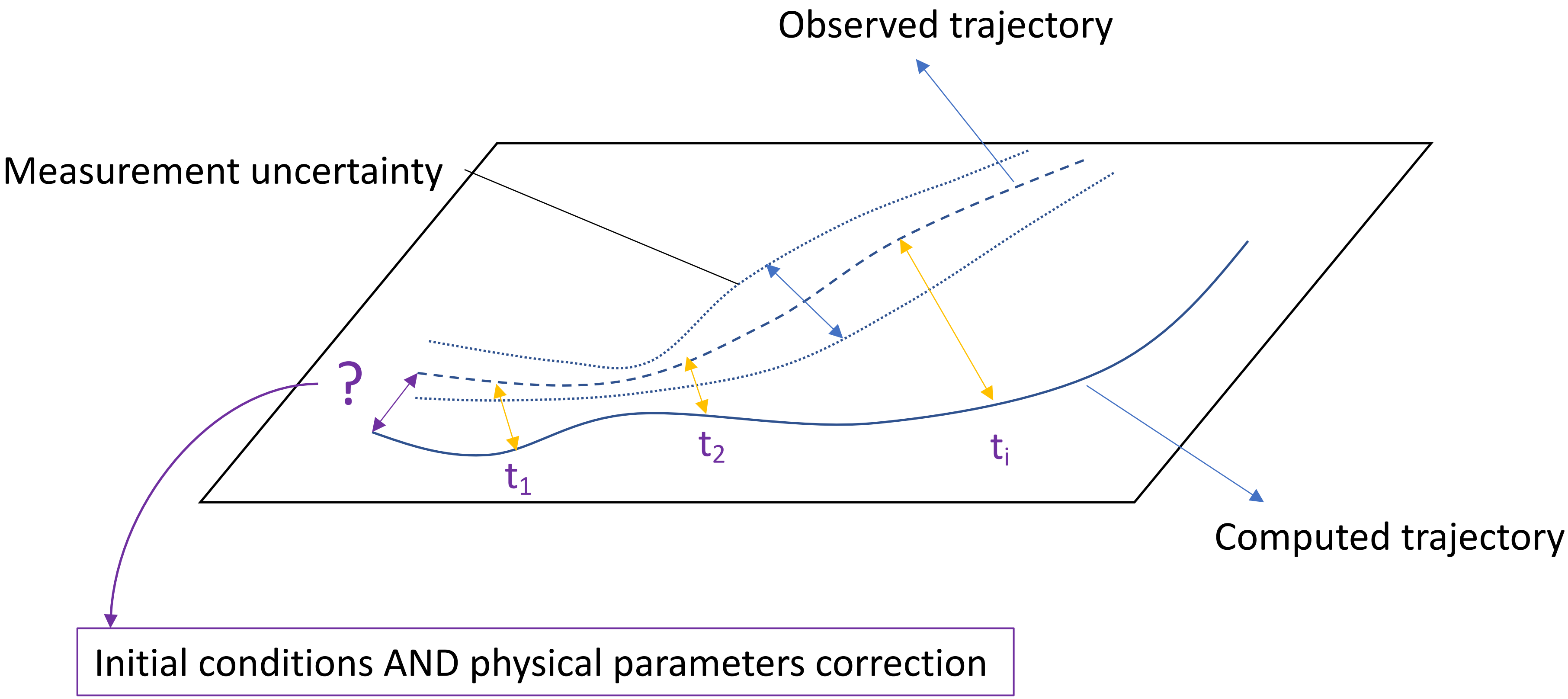
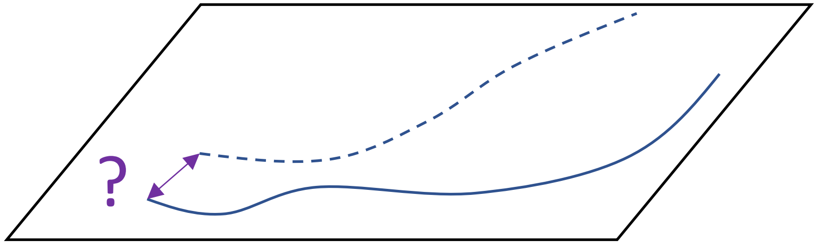


Quantification of tides from astrometry

Integration of equations of motion

$$\ddot{\vec{r}}_i = -G(m_0 + m_i)\left(\frac{\vec{r}_i}{r_i^3} - \nabla_i U_{\bar{i}\hat{0}} + \nabla_0 U_{\bar{0}\hat{i}}\right) + \sum_{j=1,\, j\neq i}^{N} G m_j \left(\frac{\vec{r}_j - \vec{r}_i}{r_{ij}^{\;3}} - \frac{\vec{r}_j}{r_j^{\;3}} - \nabla_j U_{\bar{j}\hat{i}} + \nabla_i U_{\bar{i}\hat{j}} + \nabla_j U_{\bar{j}\hat{0}} - \nabla_0 U_{\bar{0}\hat{j}}\right)$$

$$+ \frac{(m_0 + m_i)}{m_i m_0}\left(\vec{F}_{\bar{i}\hat{0}}^{\;T} - \vec{F}_{\bar{0}\hat{i}}^{\;T}\right) - \frac{1}{m_0}\sum_{j=1,\, j\neq i}^{N}\left(\vec{F}_{\bar{j}\hat{0}}^{\;T} - \vec{F}_{\bar{0}\hat{j}}^{\;T}\right) \; + GR$$



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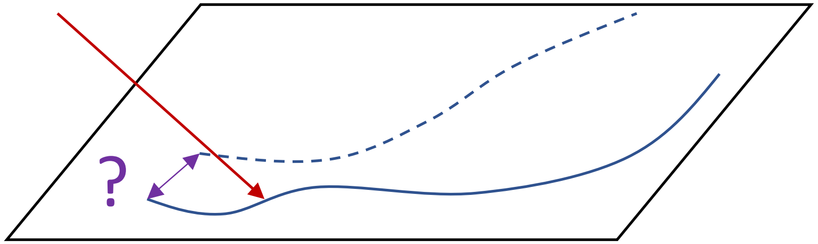

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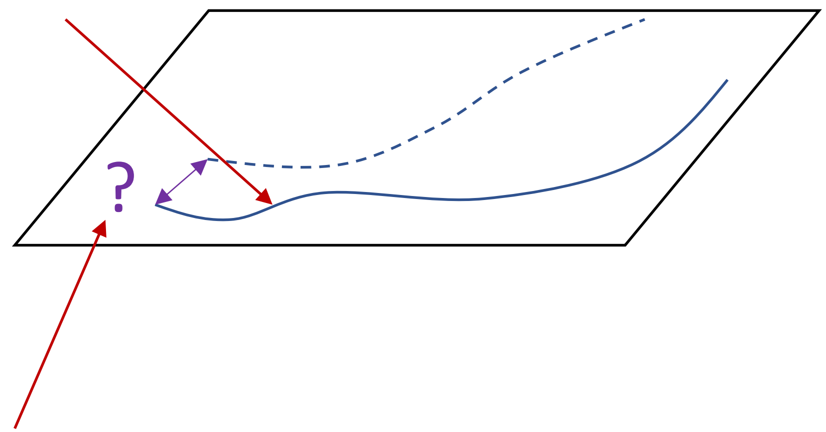
$$+\frac{(m_0+m_i)}{m_i m_0}\left(\vec{F}_{\bar{i}\hat{0}}{}^{T} - \vec{F}_{\bar{0}\hat{i}}{}^{T}\right) - \frac{1}{m_0}\sum_{j=1,\, j\neq i}^{N}\left(\vec{F}_{\bar{j}\hat{0}}{}^{T} - \vec{F}_{\bar{0}\hat{j}}{}^{T}\right) + GR$$



Simultaneously with the variationnal equations

$$\frac{d^2}{dt^2}\left(\frac{\partial \vec{r_i}}{\partial c_l}\right) = \frac{1}{m_i}\left[\sum_j \left(\frac{\partial \vec{F}_i}{\partial \vec{r}_j}\frac{\partial \vec{r}_j}{\partial c_l} + \frac{\partial \vec{F}_i}{\partial \dot{\vec{r}}_j}\frac{\partial \dot{\vec{r}}_j}{\partial c_l}\right) + \frac{\partial \vec{F}_i}{\partial c_l}\right]$$

Integration of the variational system is much more tricky and computing time consuming than equations of motion.

Quantification of tides from astrometry

Integration of the equations of motion + variational equations

observations

Taylor expansion

Approximation by a linear system
=> least squares method

$$\left(\frac{O-C}{\sigma_k}\right) = \frac{1}{\sigma_k}\sum \frac{\partial \vec{r}}{\partial c_l} \Delta c_l + \cancel{O(\Delta c_l^2)}$$

Weight of data set "*k*"

Unknowns (i.e. initial state vectors, physical parameters)

Quantification of tides from astrometry

Integration of the equations of motion + variational equations

observations

Taylor expansion

Approximation by a linear system
=> *least squares method*

$$\left(\frac{O-C}{\sigma_k}\right) = \frac{1}{\sigma_k}\sum \frac{\partial \vec{r}}{\partial c_l}\Delta c_l + \cancel{O(\Delta c_l^2)}$$

Unknowns

(i.e. initial state vectors, physical parameters)

Weight of data set "*k*"

THIS ASSUMES PERFECT MODELLING!!!

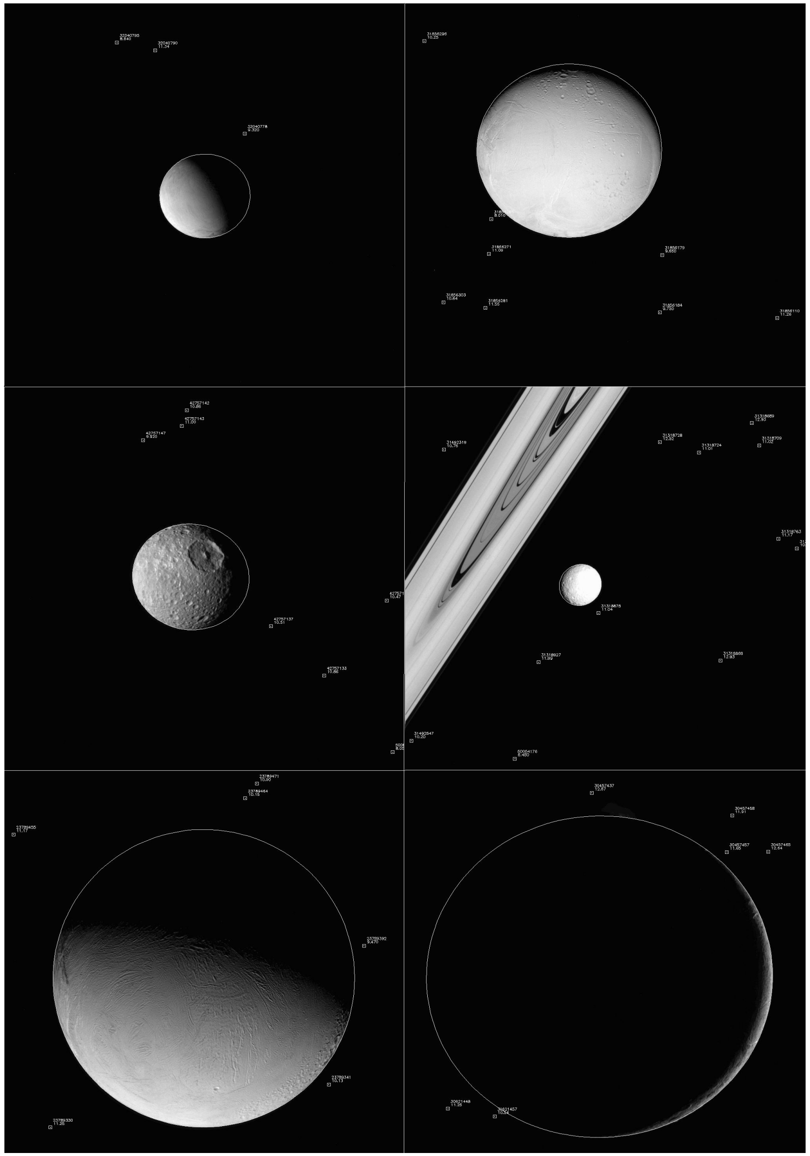
Quantification of tides from astrometry

Summary

1) Get as much astrometric data as possible
2) Model the dynamics of the planetary system
3) Integrate the equations of motion and variational equations simultaneously
4) Solve the linear system using an inversion method
5) Iterate the procedure few/several times until full convergence

Quantification of tides in Saturn:

Saturnian tidal dissipation from astrometry suggests tides at least **10x** stronger



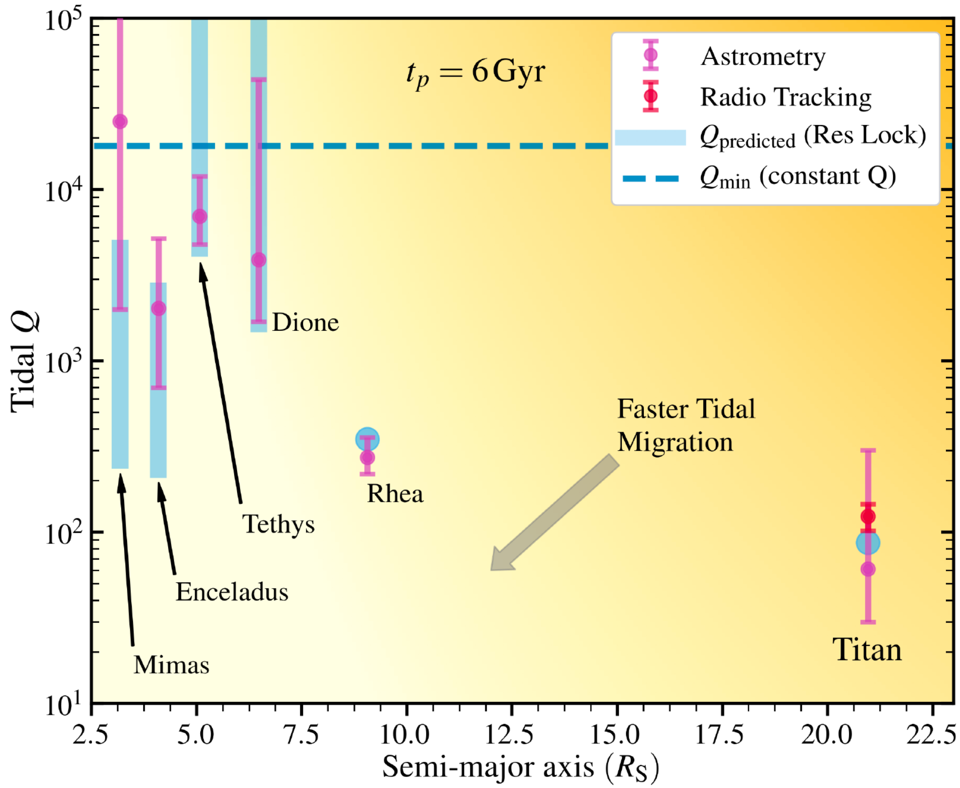
Tajeddine et al. 2013 - Cassini NAC-ISS

$$\ddot{\vec{r}}_i = -G(m_0+m_i)\left(\frac{\vec{r}_i}{r_i^3} - \nabla_i U_{\bar{i}\hat{0}} + \nabla_0 U_{\bar{0}\hat{i}}\right) + \sum_{j=1,\, j\neq i}^{N} G m_j \left(\frac{\vec{r}_j - \vec{r}_i}{r_{ij}^3} - \frac{\vec{r}_j}{r_j^3} - \nabla_j U_{\bar{j}\hat{i}} + \nabla_i U_{\bar{i}\hat{j}} + \nabla_j U_{\bar{j}\hat{0}} - \nabla_0 U_{\bar{0}\hat{j}}\right)$$

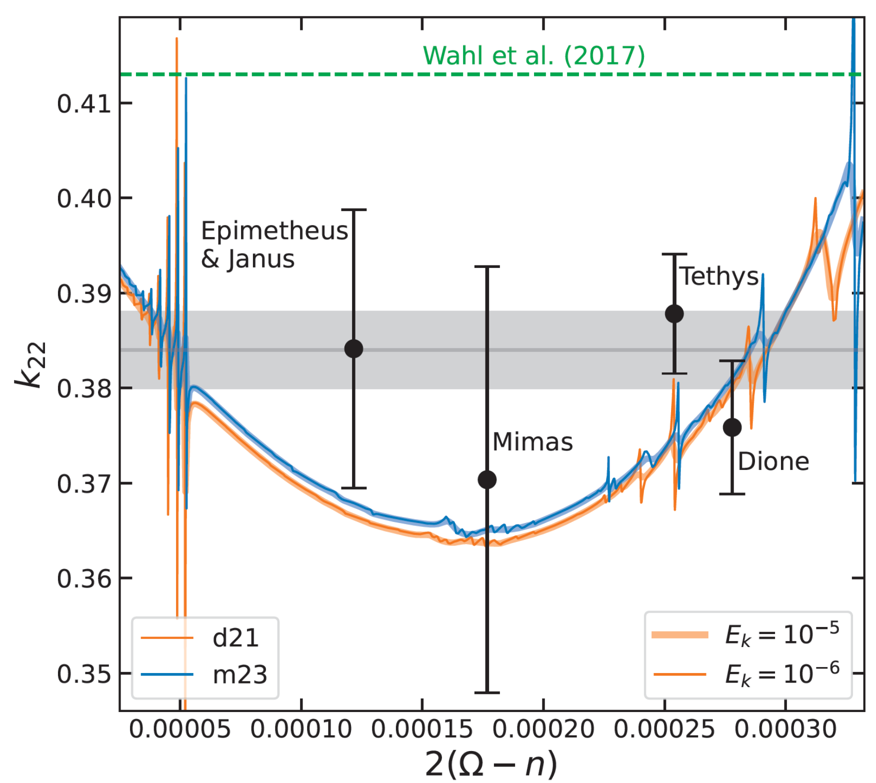
$$+\frac{(m_0+m_i)}{m_i m_0}\left(\vec{F}_{\bar{i}\hat{0}}^{\,T} - \vec{F}_{\bar{0}\hat{i}}^{\,T}\right) - \frac{1}{m_0}\sum_{j=1,\, j\neq i}^{N}\left(\vec{F}_{\bar{j}\hat{0}}^{\,T} - \vec{F}_{\bar{0}\hat{j}}^{\,T}\right) \quad (1)$$

Q=1682 +/-540 (Lainey et al. 2012)

Quantification of tides in Saturn:



Lainey et al. 2020



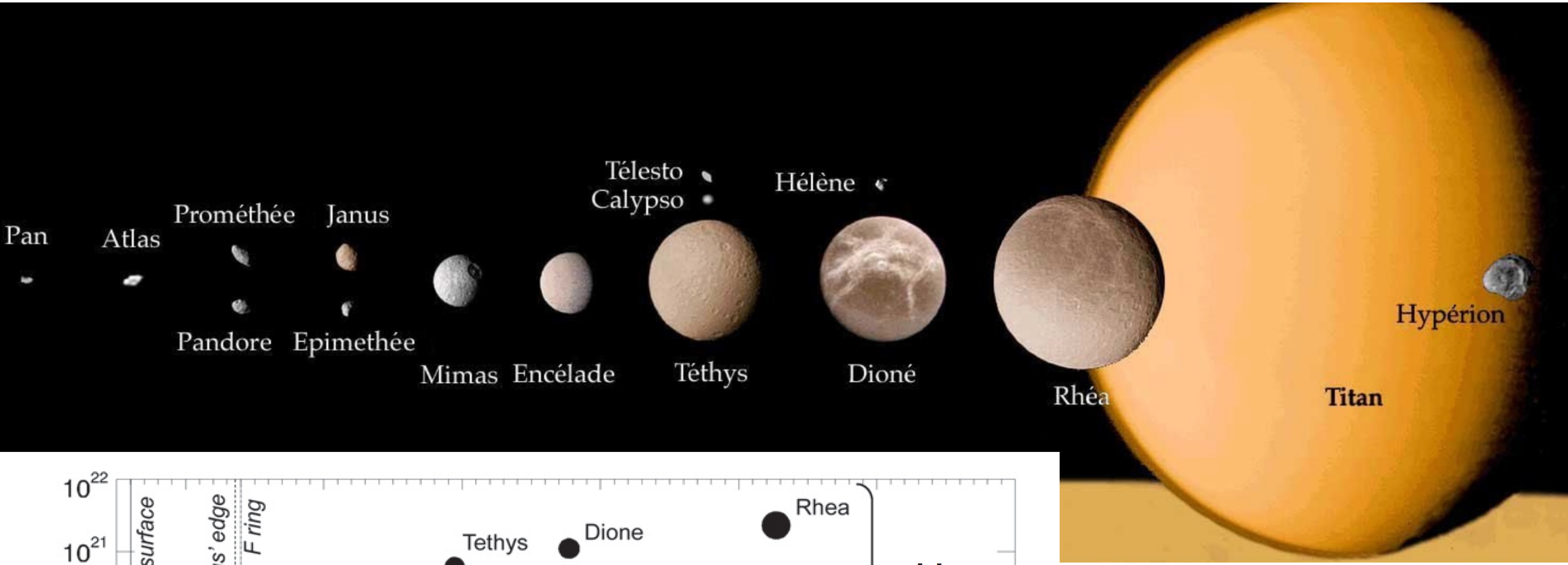
Lainey et al. 2024

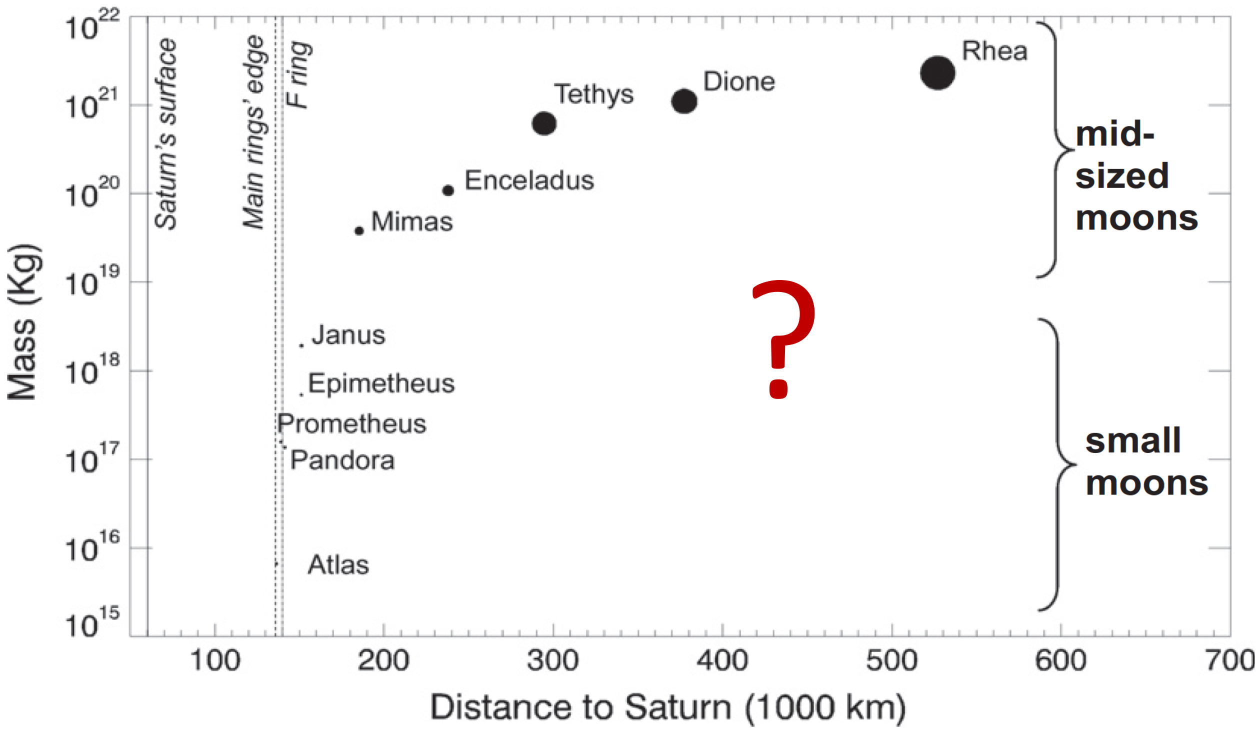
Cassini astrometry data allow to solve for k_2 and Q at four and six tidal frequencies!

Part III

Asnwering question 1

The relative size of the main moons – Question 1





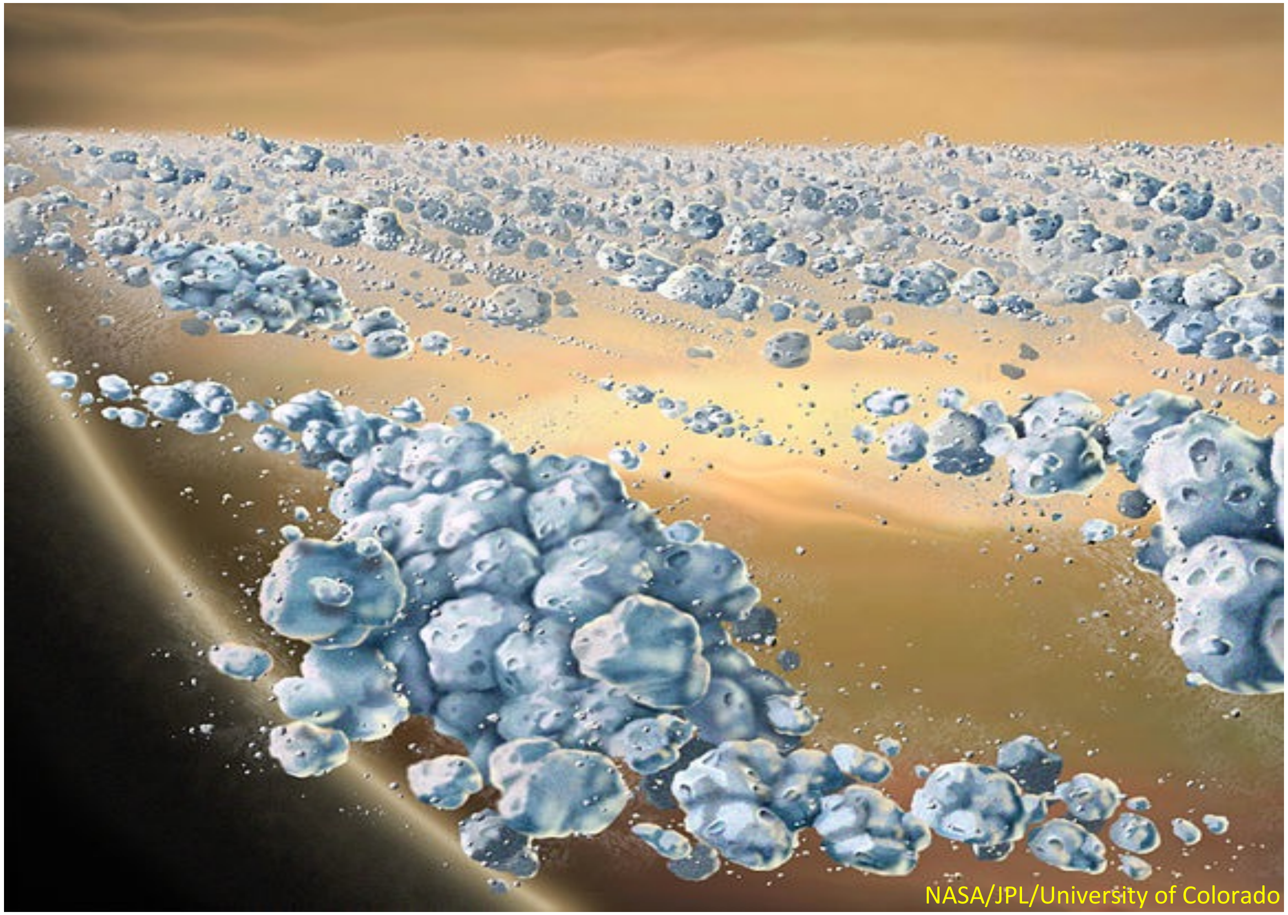
Charnoz et al. 2011

The relative size of the main moons – Question 1

Strong tidal dissipation in Saturn is **in contradiction** with a formation of the moons 4.5 Byr ago

→A formation process that occurs much later in the history of the Solar system must be considered…

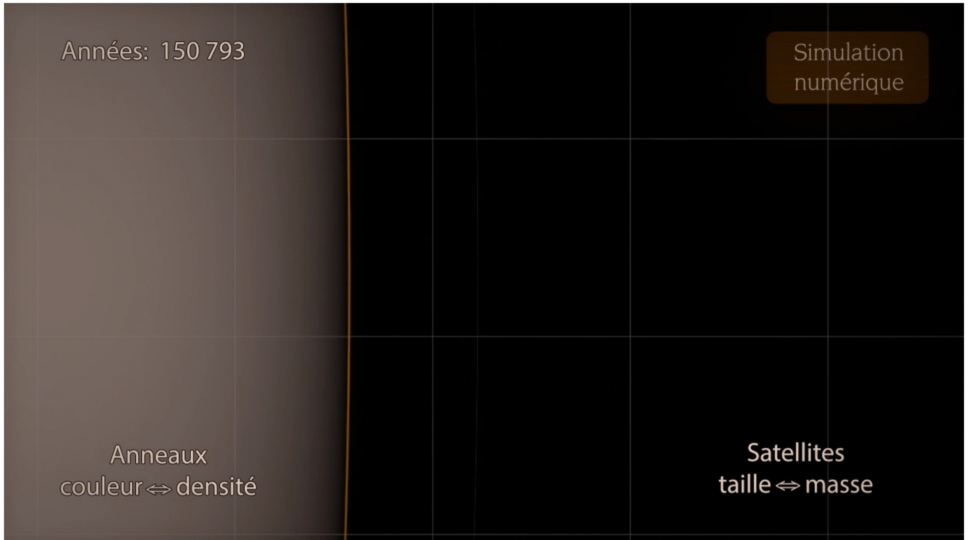
The relative size of the main moons – Question 1

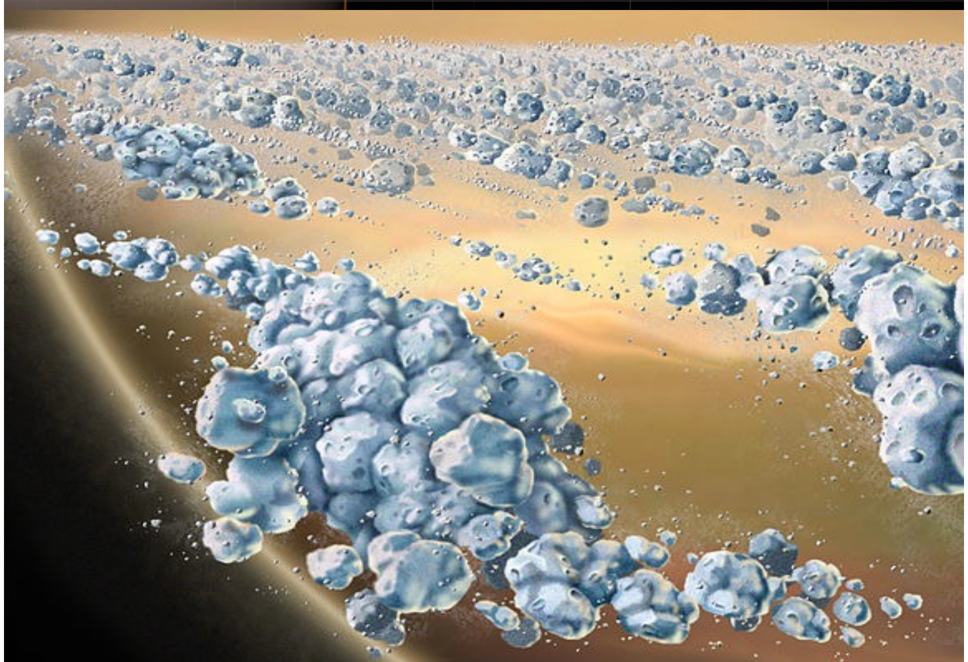


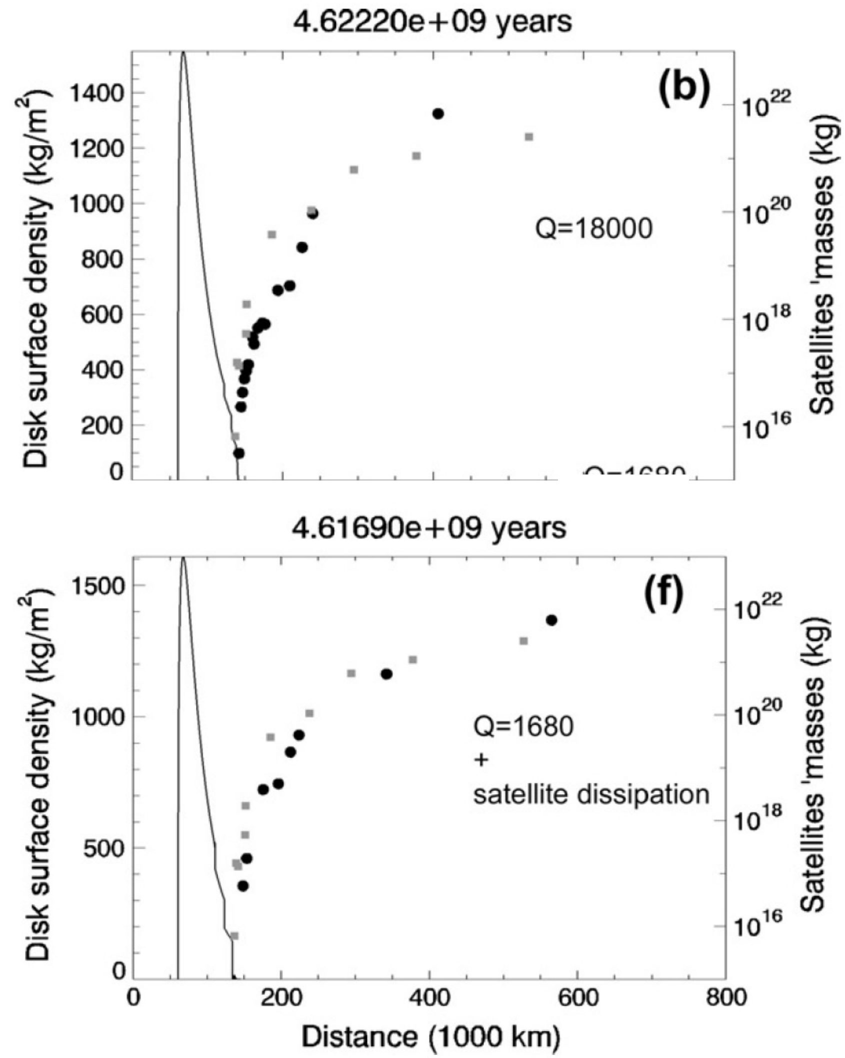
Idea: try forming the moons from an initial massive ring at the edge of the Roche limit

Saturnian Roche limit ≈ Edge of the A ring

<u>An explanation to the mass distance distibution</u>



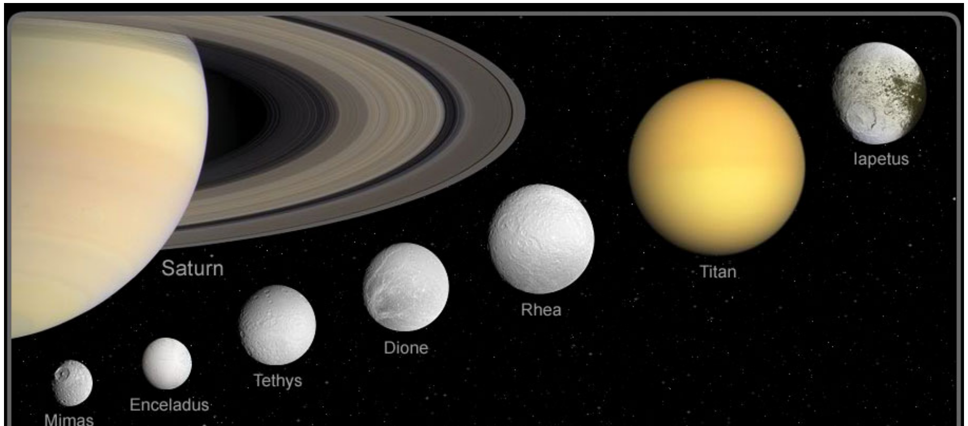


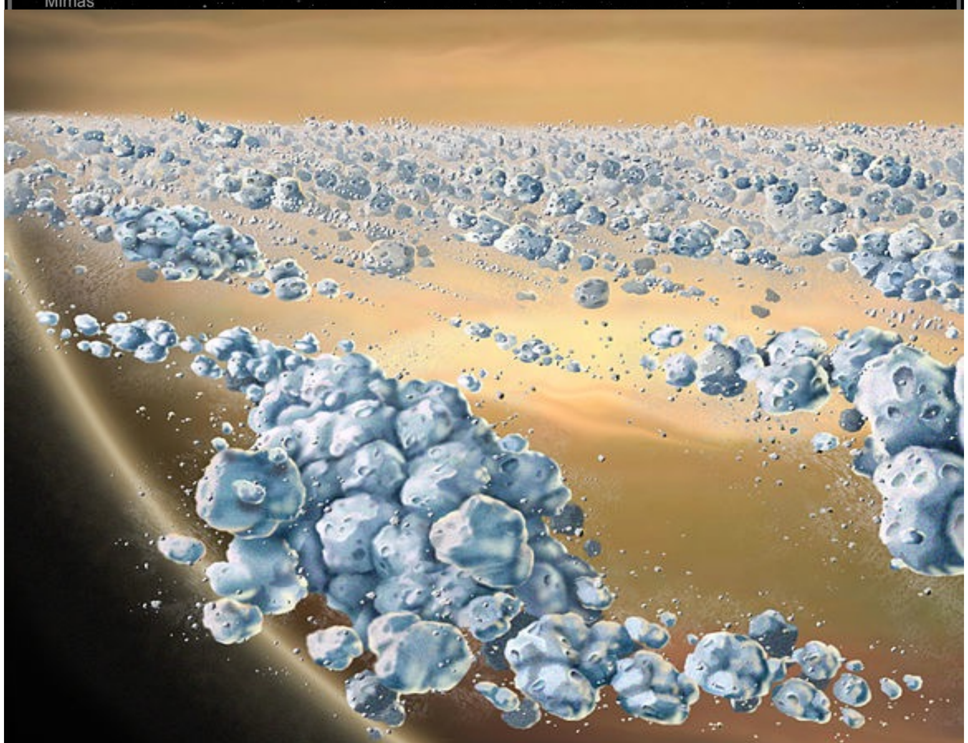


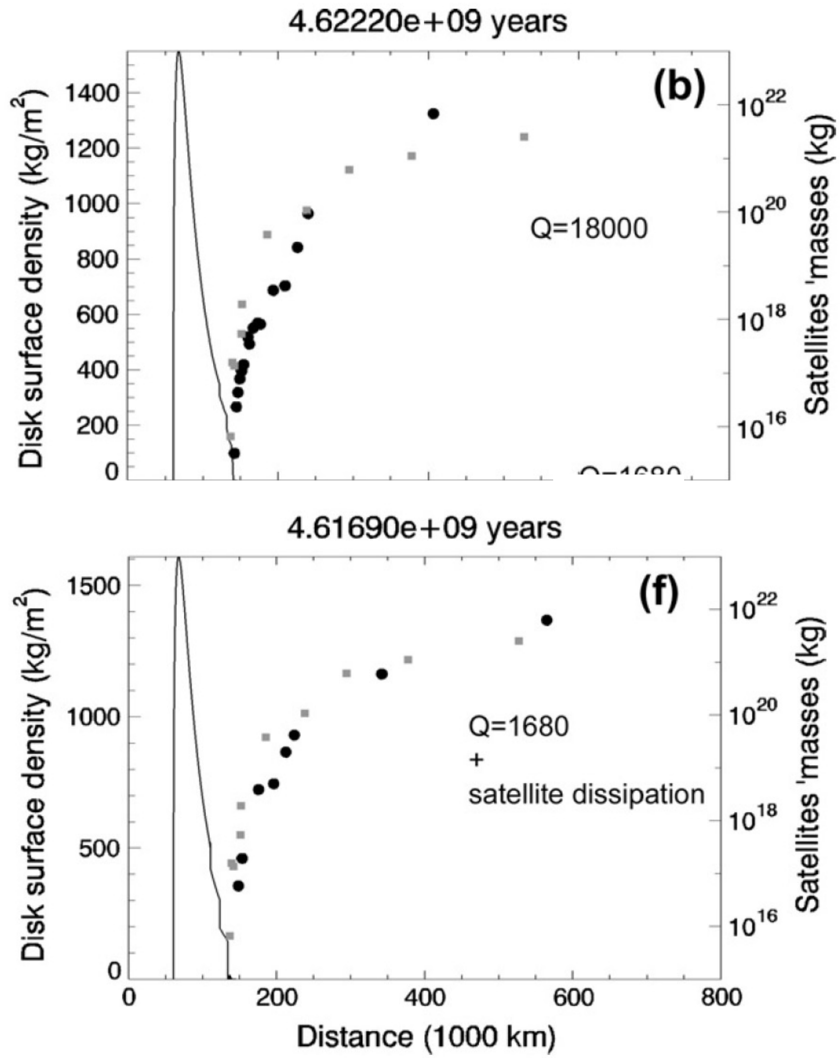
Charnoz et al. 2011, Crida and Charnoz 2012

Thanks to the strong Satunian tides we can explain the current positions of main moons after formation at the Roche limit…

An explanation to the mass distance distibution







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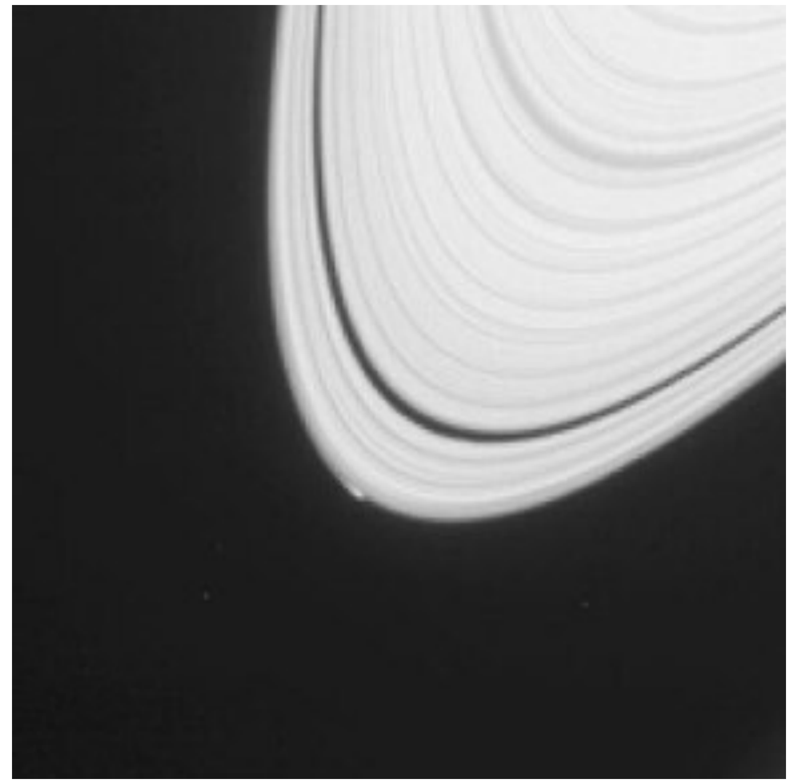
An explanation to the mass distance distibution

Discovery of « Peggy » in 2013 on ISS image

C.D.Murray (QMUL)

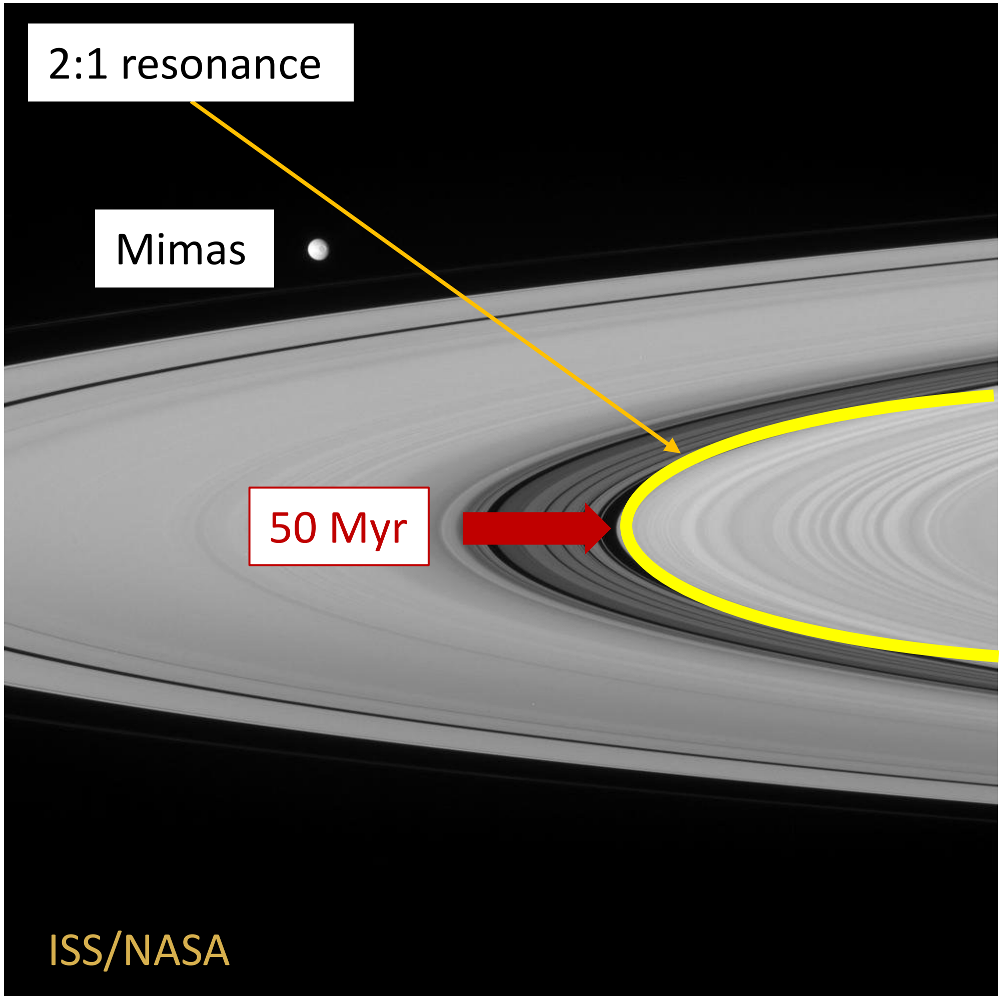


→ The suggested mecanism for producing moons may still be operating today!

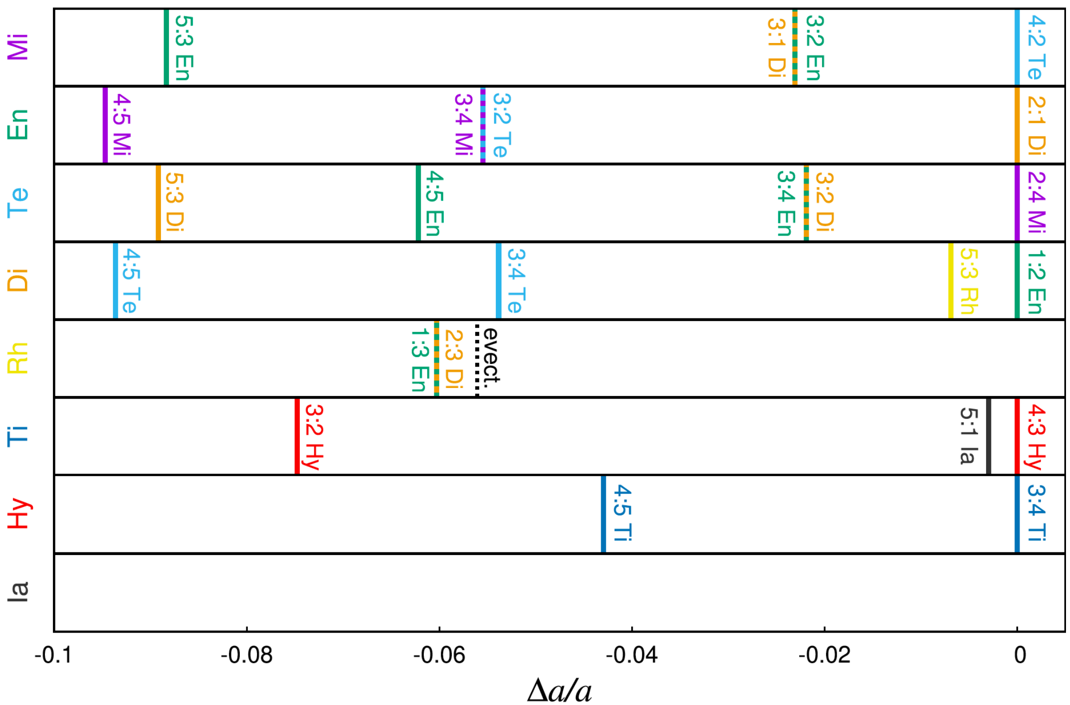
Part IV

Asnwering question 2

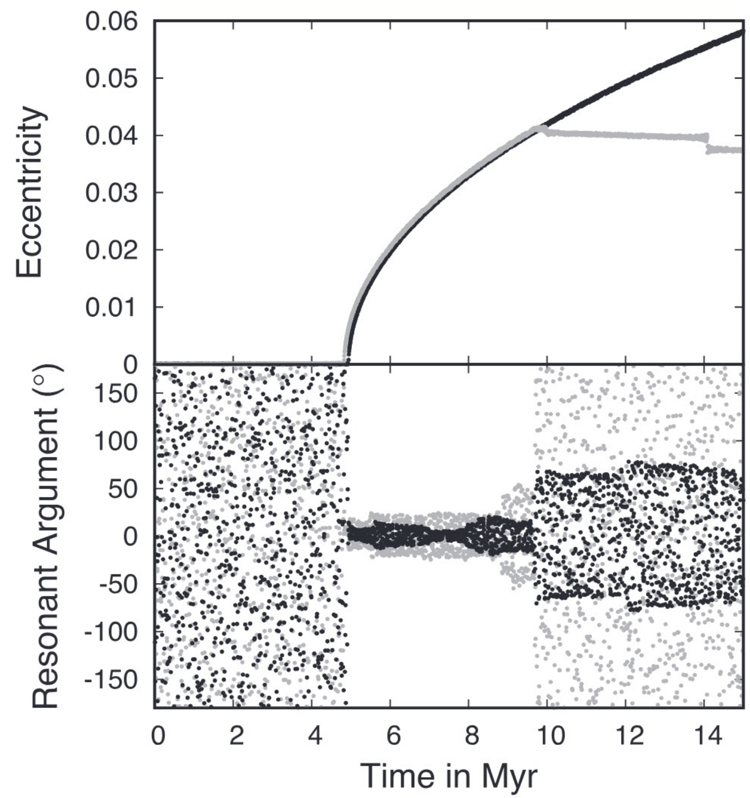
The Cassini division – Question 2



Explaining the Cassini division



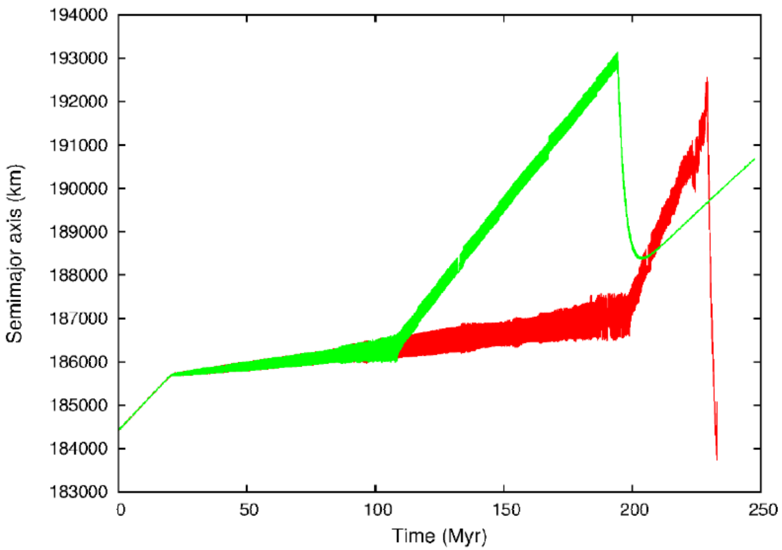
Cuk et al. 2024

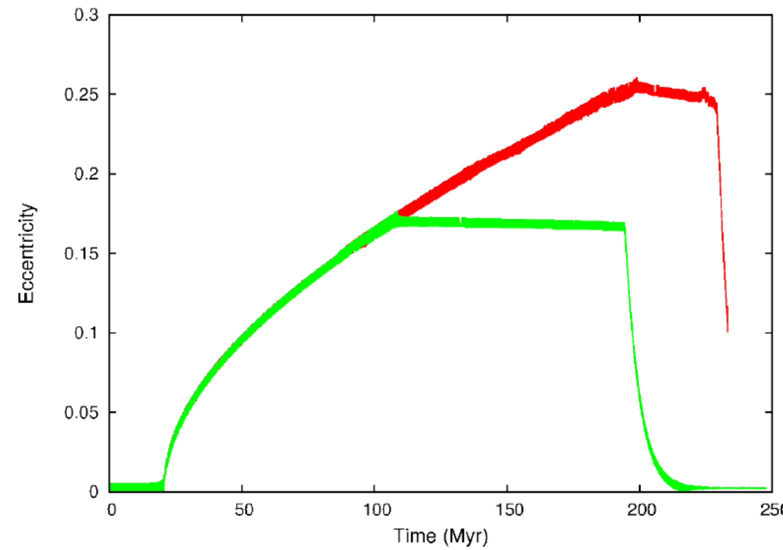


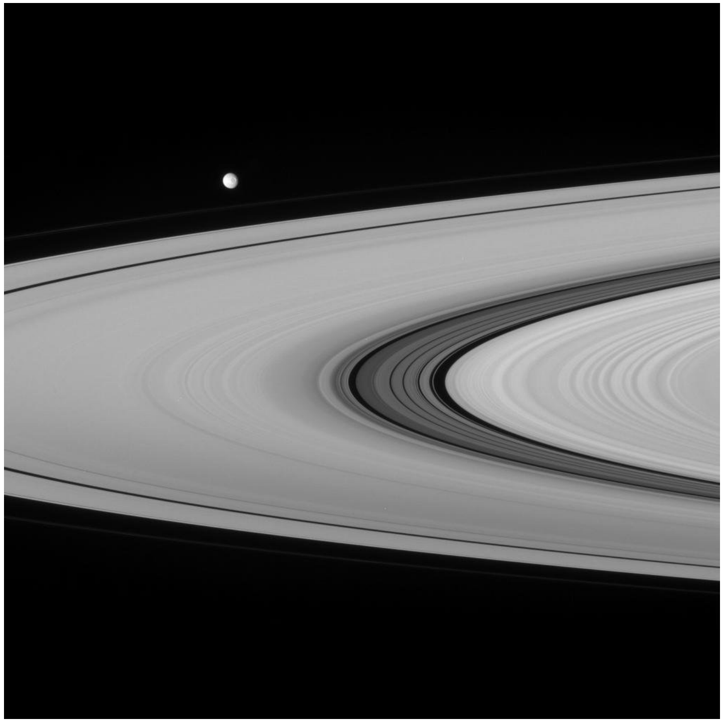
Cuk and El Moutamid 2022

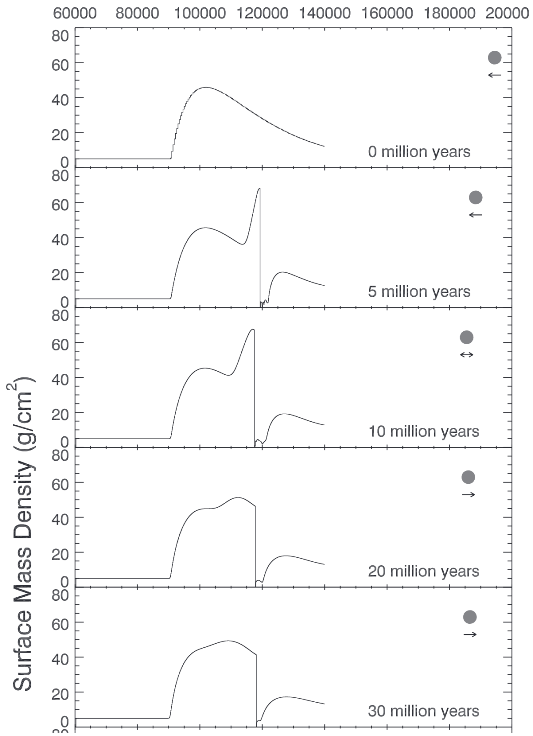
Strong tides imply fast evolution!

Explaining the Cassini division





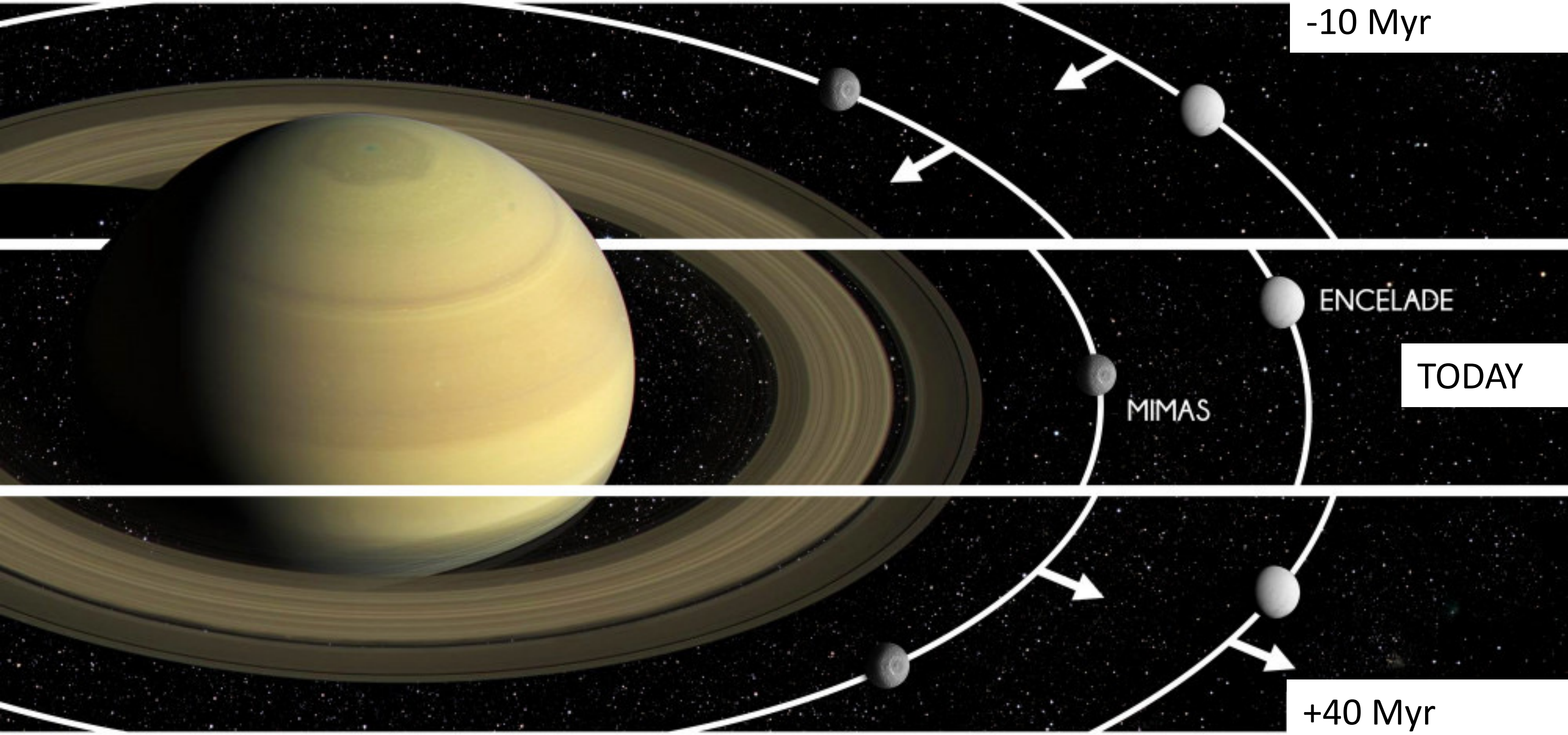




Baillié et al. 2019

Noyelles et al. 2019

Explaining the Cassini division

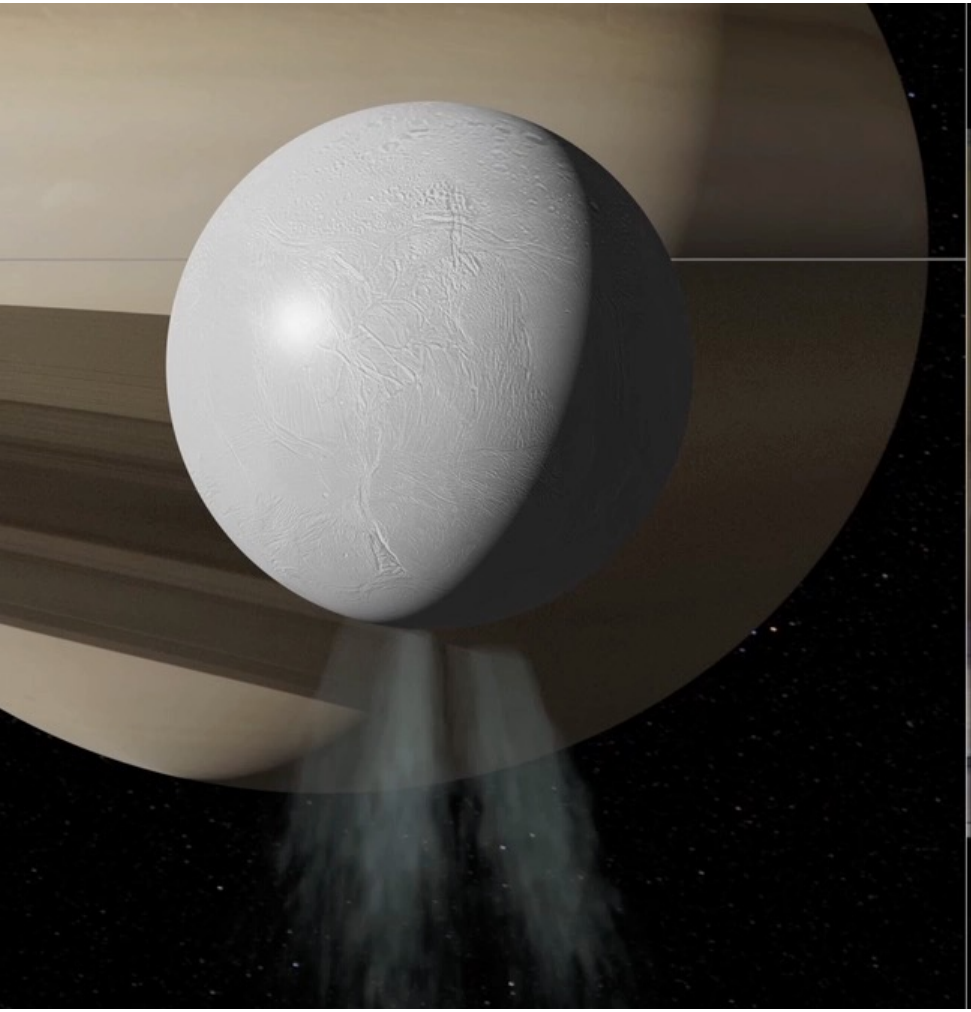


Baillié et al. 2019; Noyelles et al. 2019

Part V

Asnwering question 3

The Mimas / Enceladus paradox – Question 3



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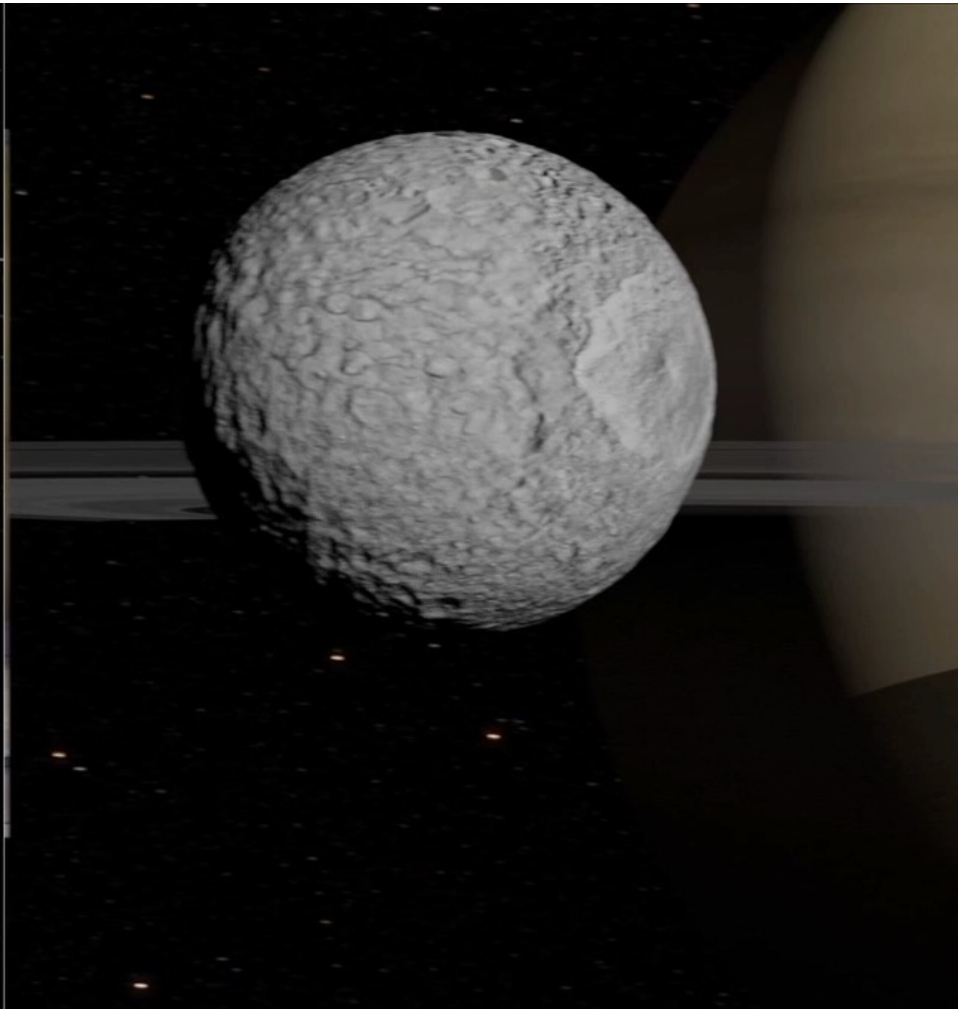

Mimas

R = 198.2 km
a = 185,539 km
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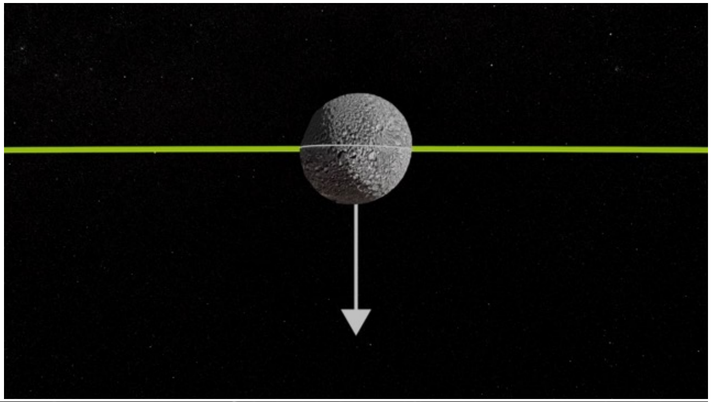
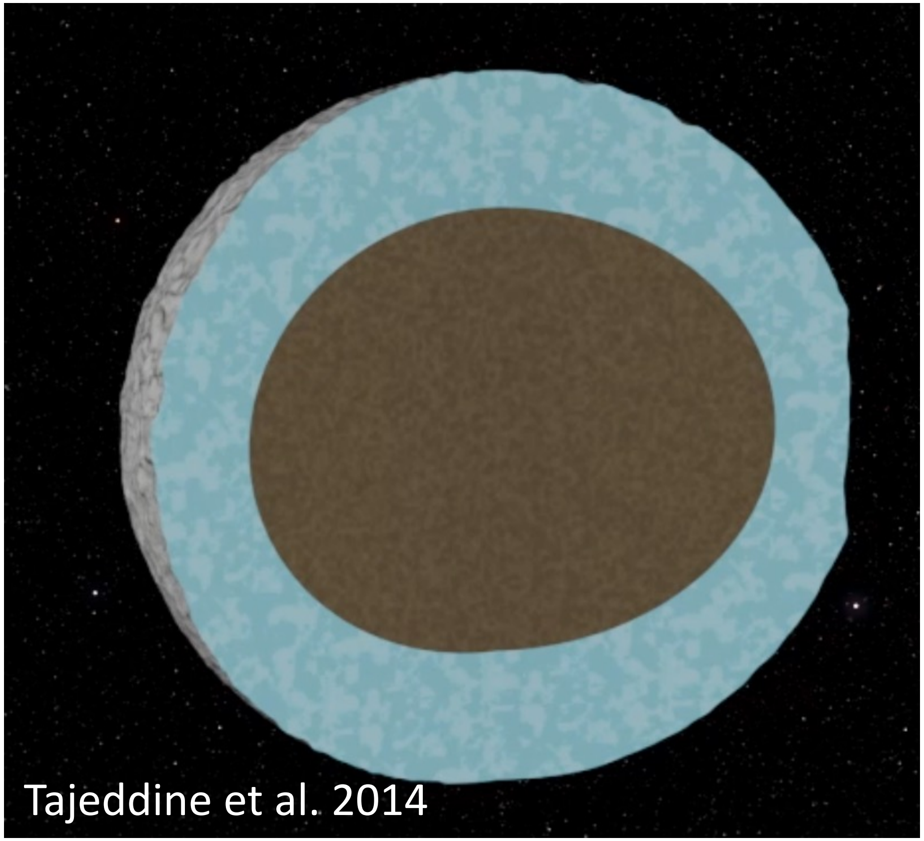
The Mimas / Enceladus paradox – Question 3

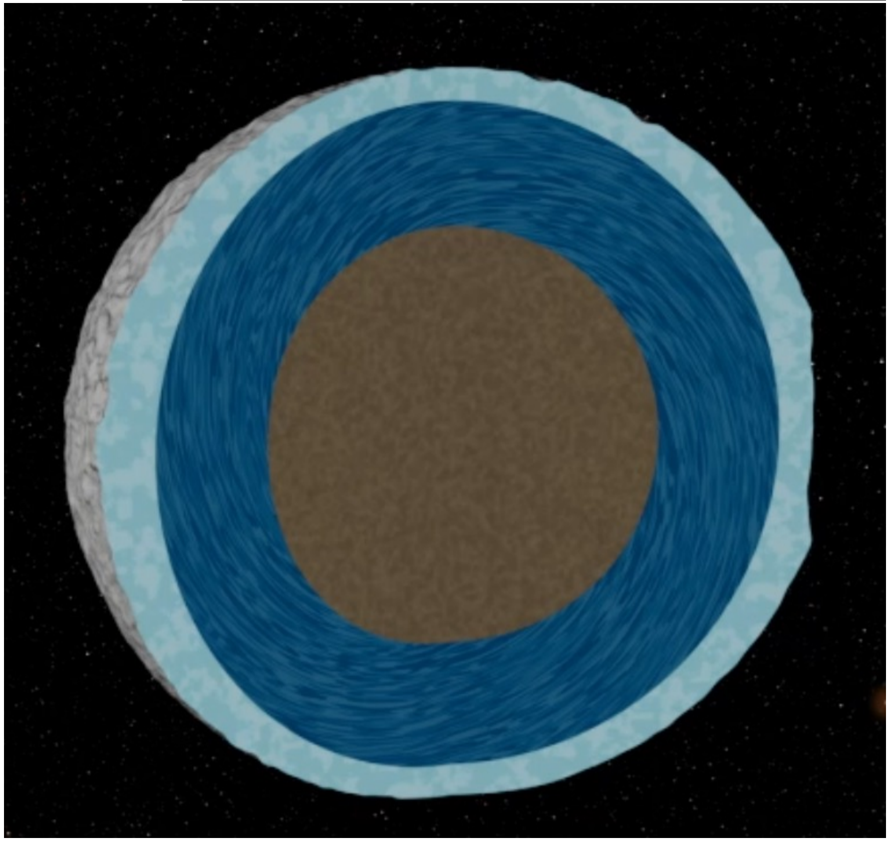
The amplitude $\boldsymbol{\varphi}$ of the physical libration of a spin-orbit moon can be related to the moments of inertia by:

$\boldsymbol{\varphi} = 2e/(1-1/3\boldsymbol{\gamma})$ with $\boldsymbol{\gamma}=(B-A)/C$

(Comstock & Bills 2003)



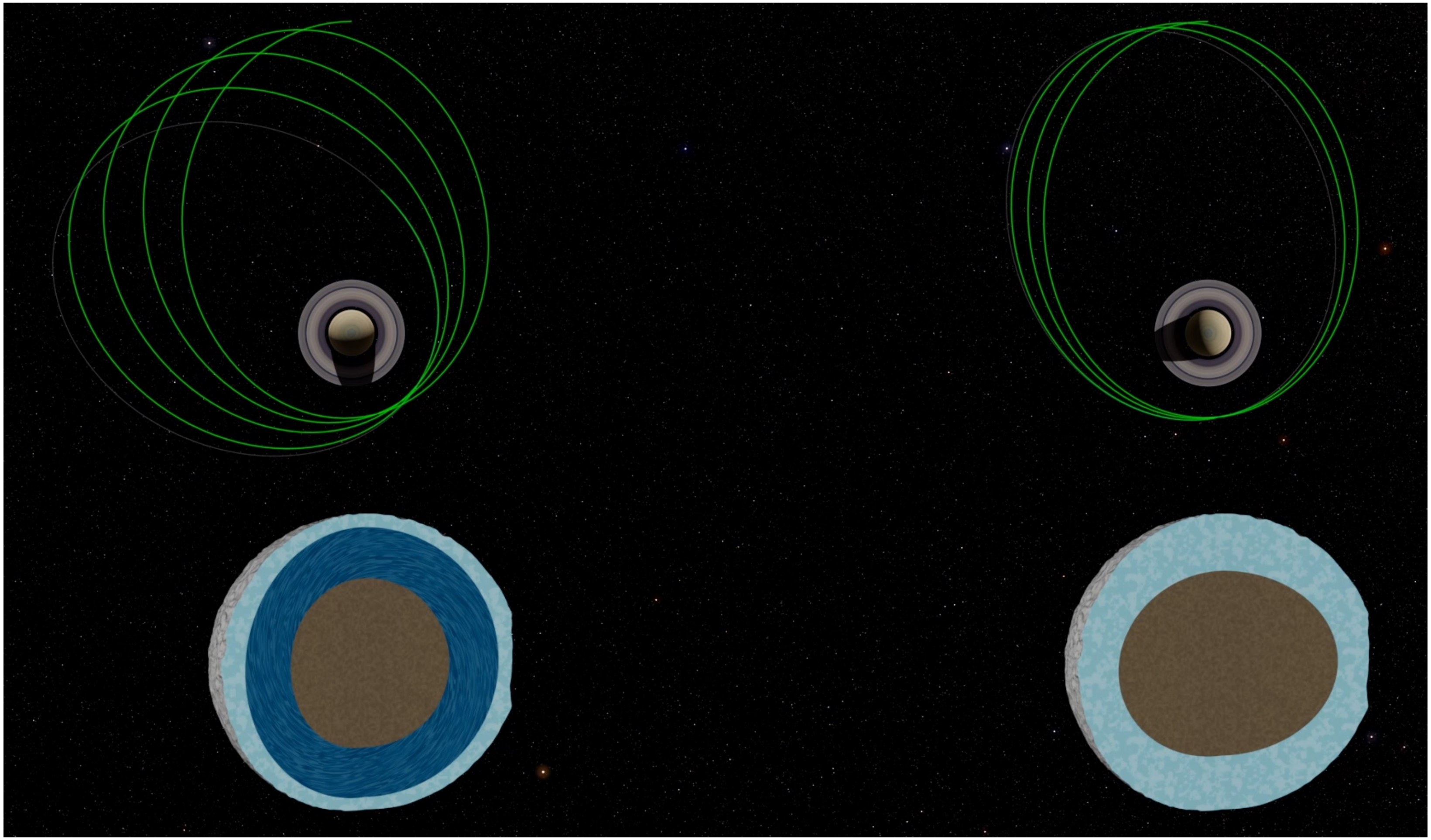




The Mimas / Enceladus paradox – Question 3

To discriminate between both interior model, one needs an extra information

→Using the feedback of the rotation on the orbital motion



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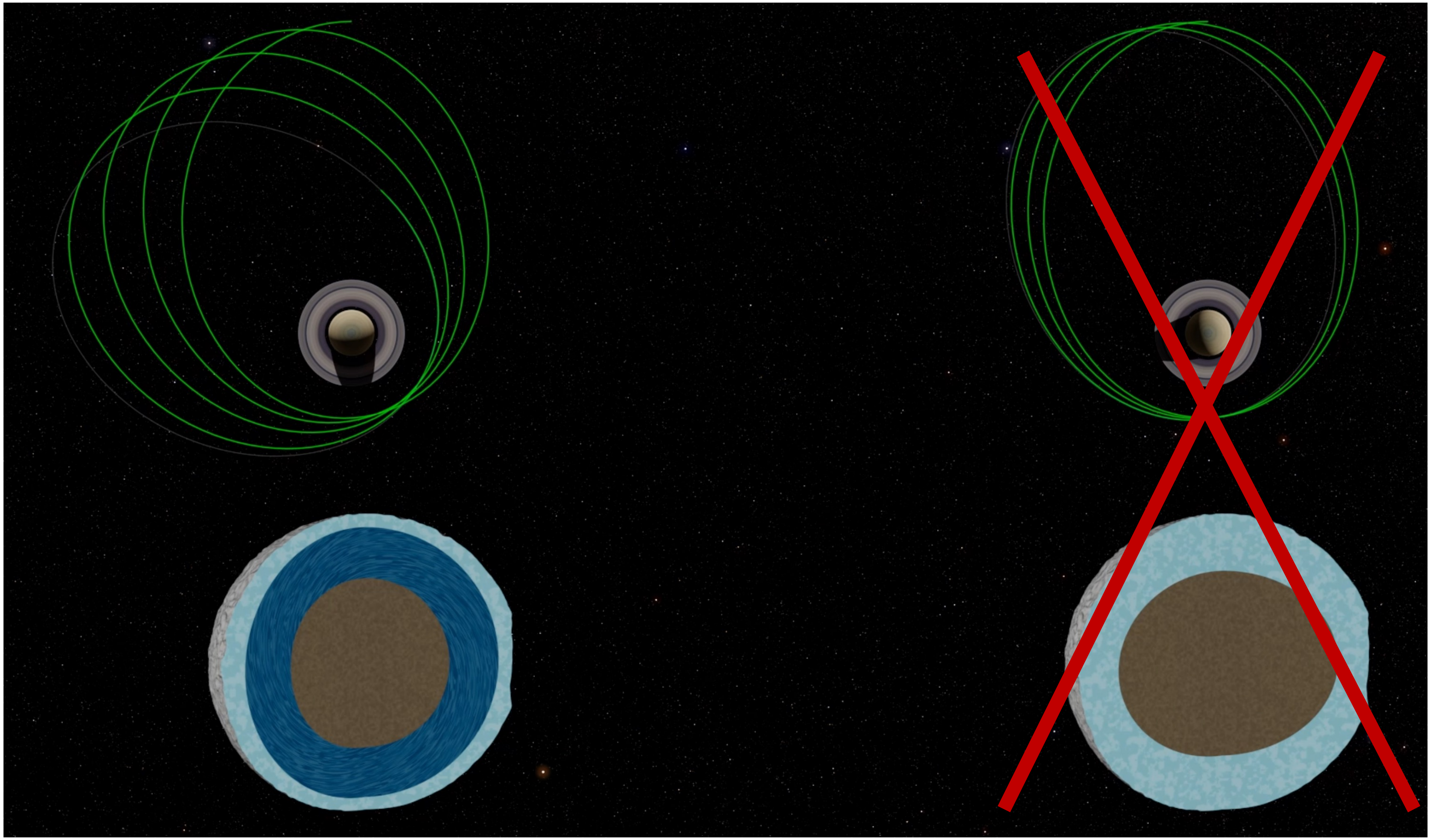
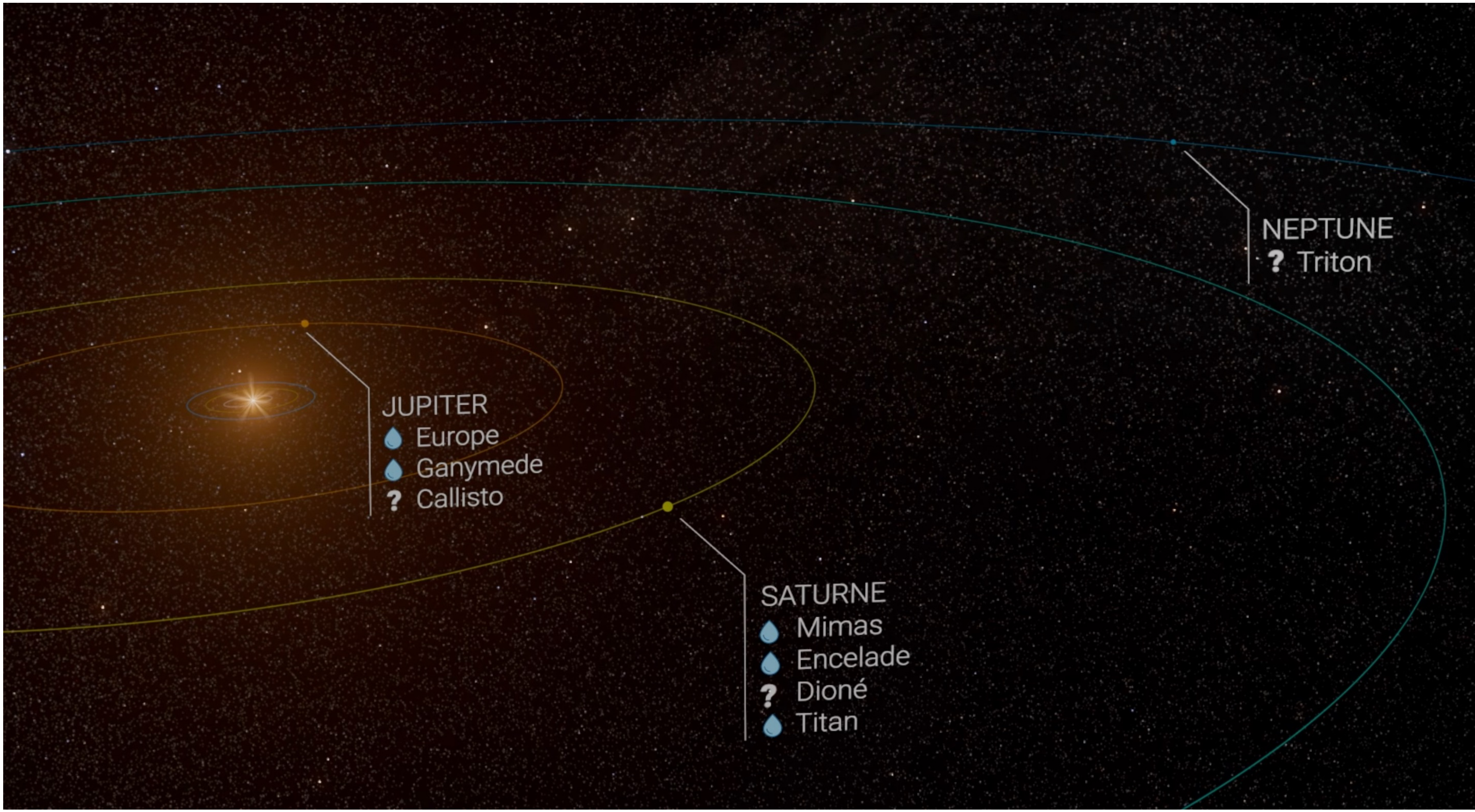
To discriminate between both interior model, one needs an extra information

→Using the feedback of the rotation on the orbital motion



Lainey, Rambaux, Tobie et al. 2024

Conclusion -“and beyond…”



Conclusion -“and beyond…”